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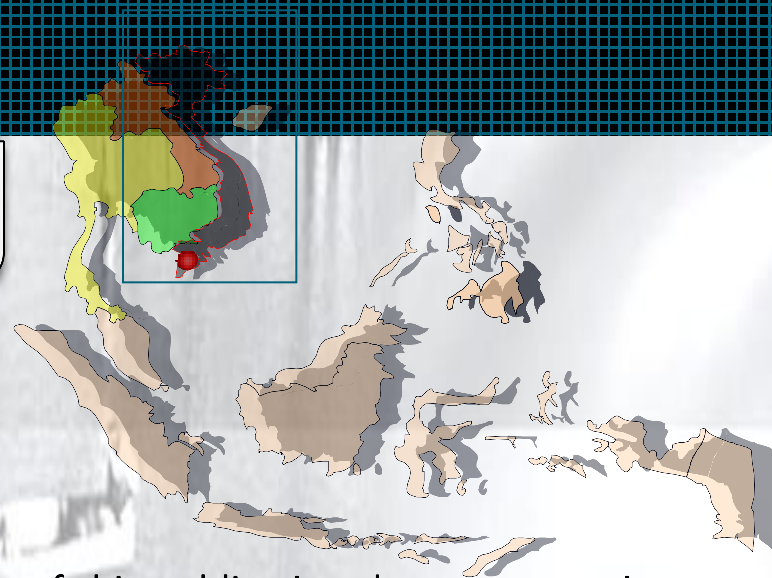
ENVIRONMENTAL MODELLING



Lecturer:
Dr. Huynh Vuong Thu Minh

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Part 1

CONTENTS

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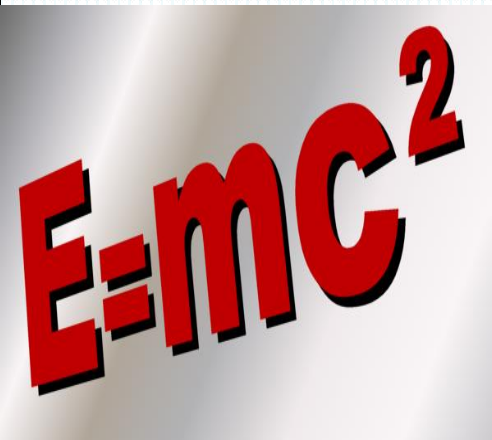
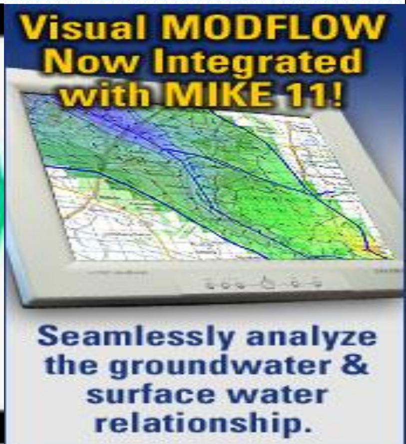
INTRODUCTION

Part 1

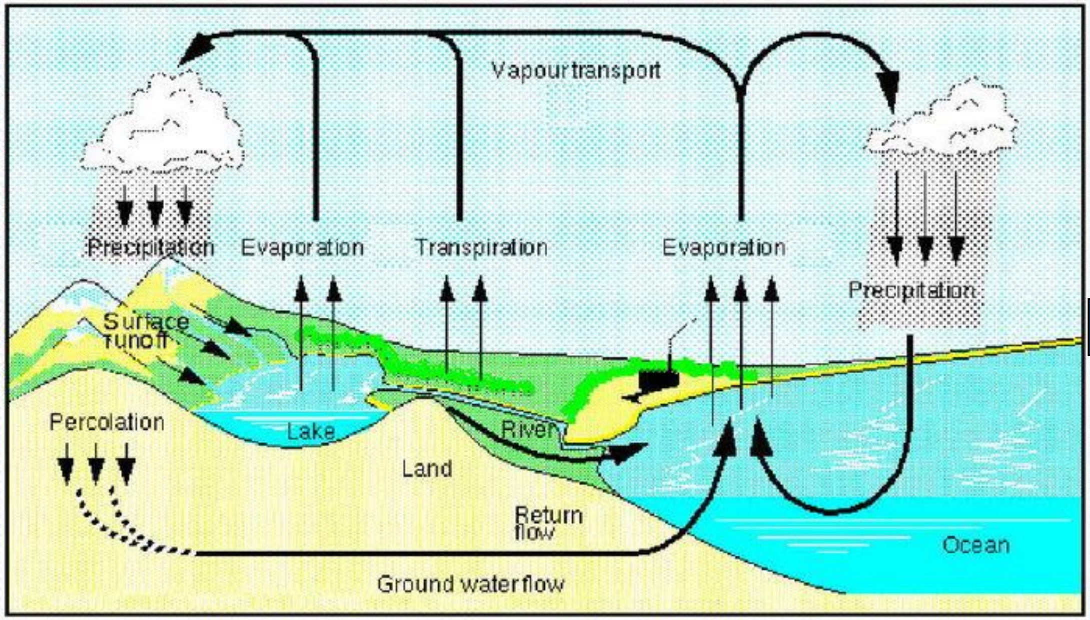
1. Model definition

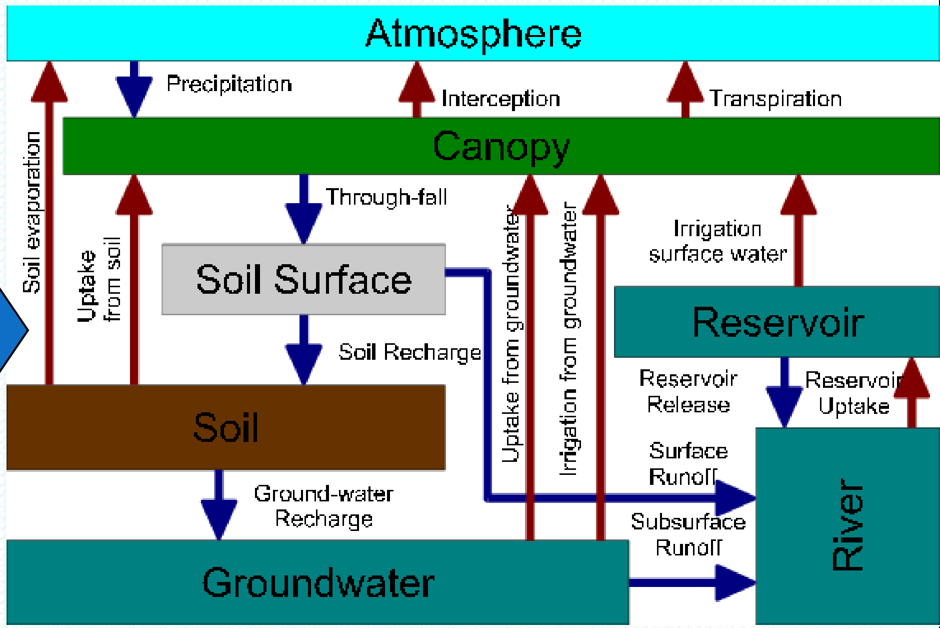
✓ Models might be specific objects (mathematics or equations), systems, or concepts (thoughts) that replace the original (Claude Shannon, 1948).





1. Model definition





Hydrological system

Hydrological concept modelling

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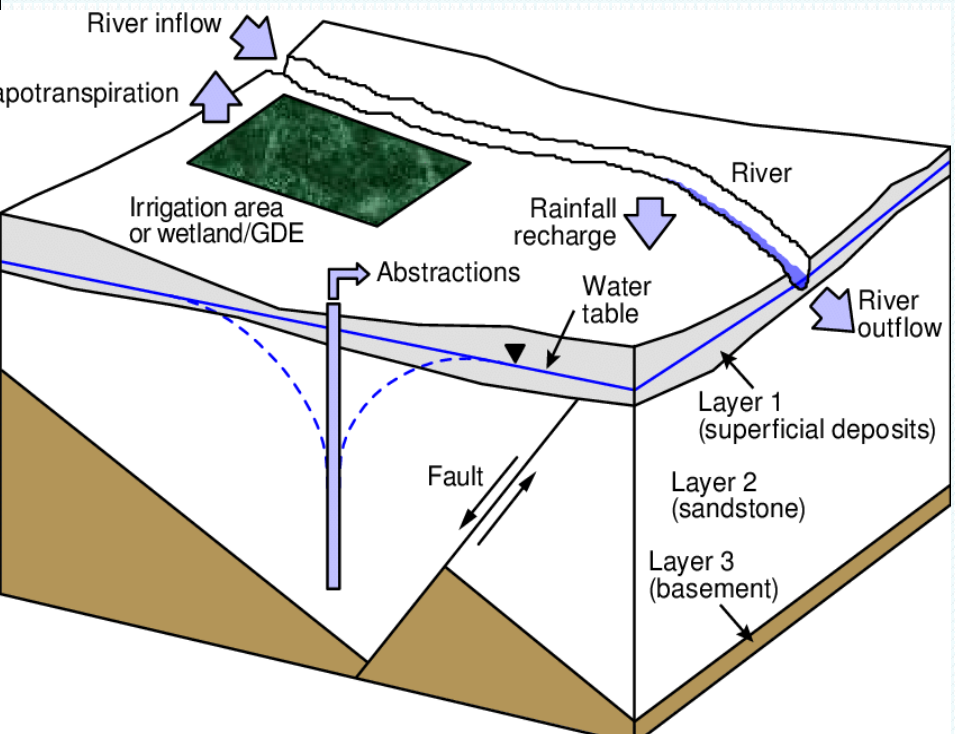
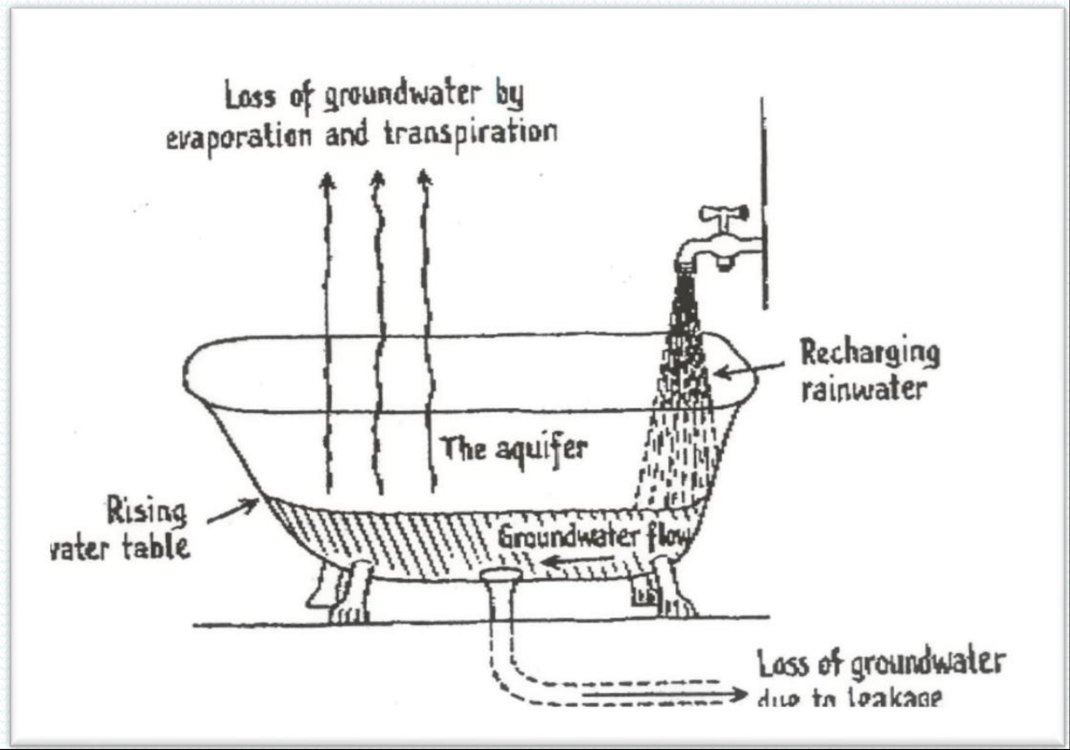
1. Model definition

✓ Models are substitutes for natural systems for natural systems (but only essential processes);





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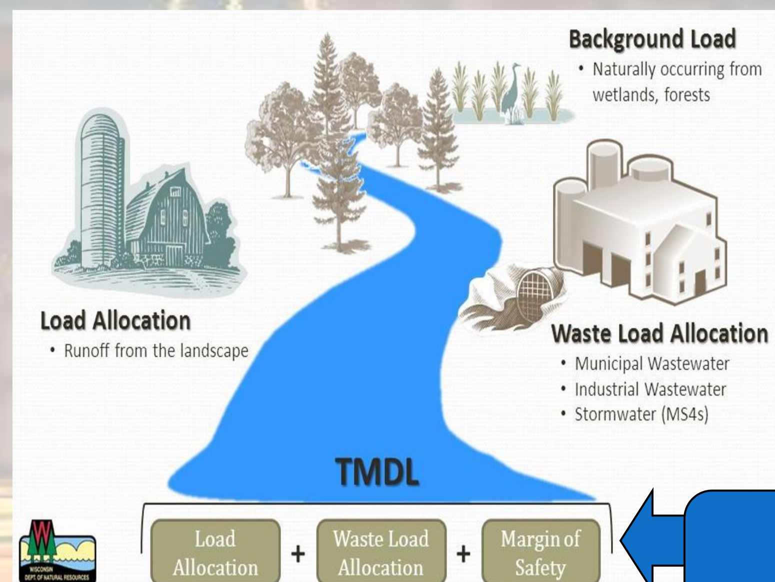
1. Model definition

- ✓ A model is a tool for testing hypotheses and doing quantitative research for decision-making;
- ✓ it is also a critical legal tool for managing, protecting, and resolving consequences.



Safety threshold

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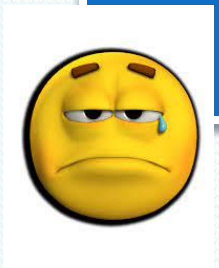
Principles of Modelling



- The best models are connected to reality;
- Every model may be expressed at different levels of precision;
- Always requires simplification;
- No single model is sufficient;
- ….

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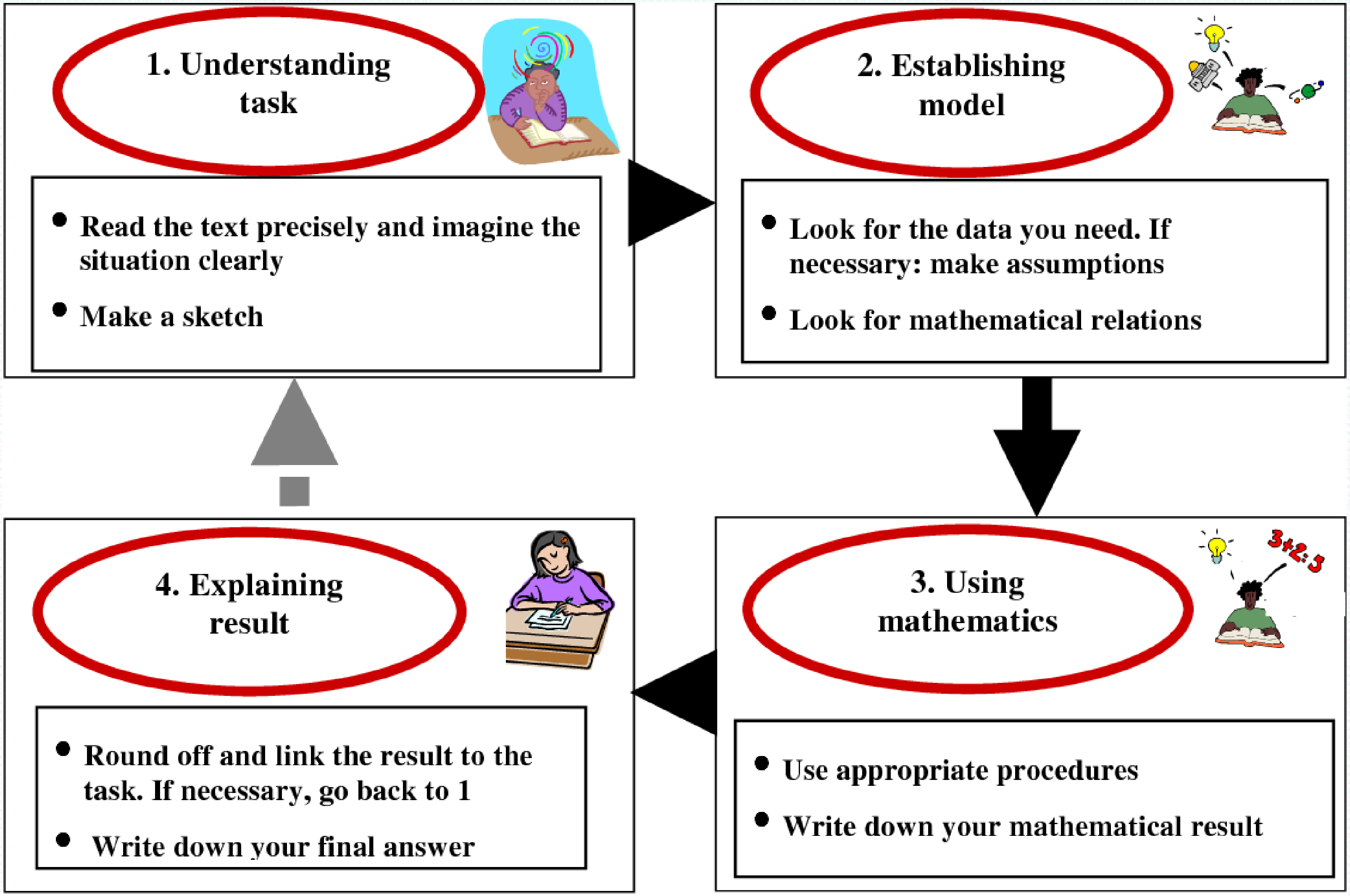
Mathematical models are used in quantitative analysis to make decisions



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Part 1

2. The necessity of model

- To better understand the system we are developing
- To visualize a system;
- Provide a “framework” for data aggregation;
- Better understanding of complex interactions (what happened?);
- Forecast scenarios (optimal choice - what's best?);
- Communicate information/visualize.

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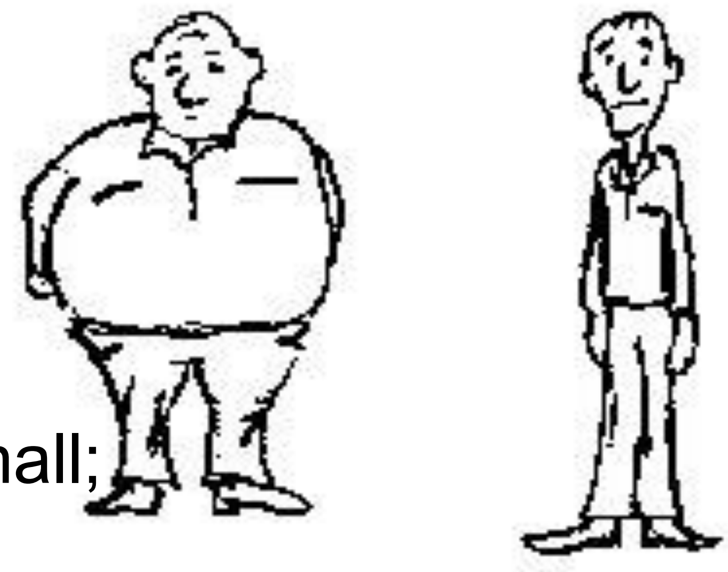
3. Purpose of environmental modelling

- To identify processes dominating water quality issues and magnitude of problems;
- To develop and evaluate management strategies; and
- To monitor compliance with water quality objectives.



4. Scale of model

- ✓ Space-scale area: large or small;
- ✓ Time-scale: short or long;
- ✓ Validity of the model: The model has been pre-set with the scale of space and time.
- ✓ Do not apply outside of the above setting.

5. Model classification

Part 1

Catwalk models

prototype house

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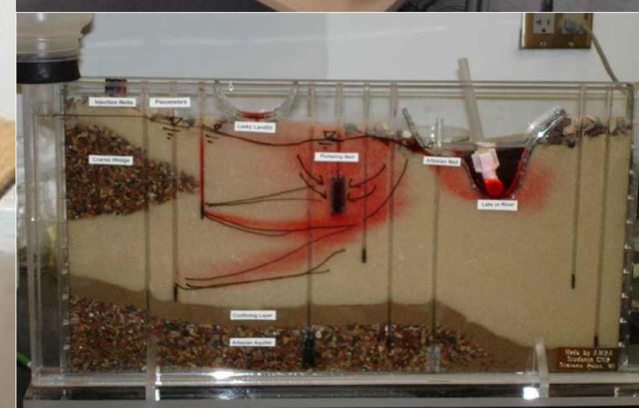
5. Model classification

5.1. scale physical modelling and prototype modelling



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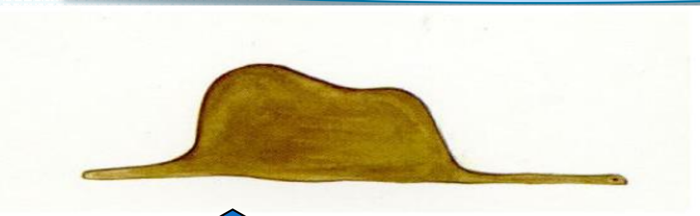
5.2. Mathematical modelling

Mathematical modelling

Uses mathematical equations to describe a system

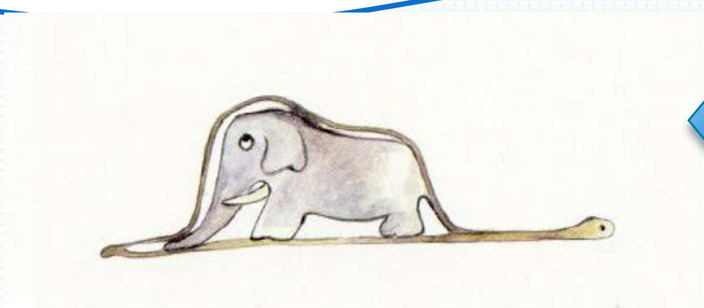


From: The little Prince

Models allow us to find structure in complex systems and to investigate how different factors interact

Numerical modelling

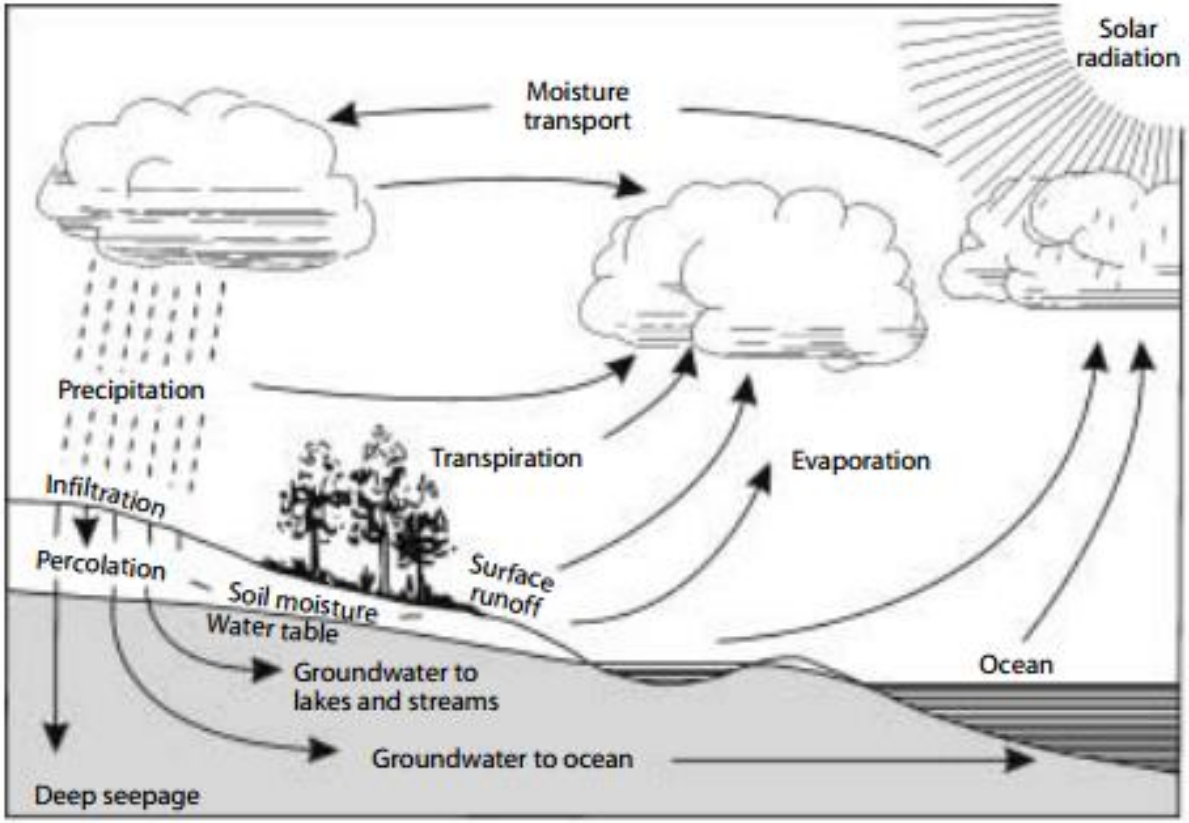
Conceptual modelling

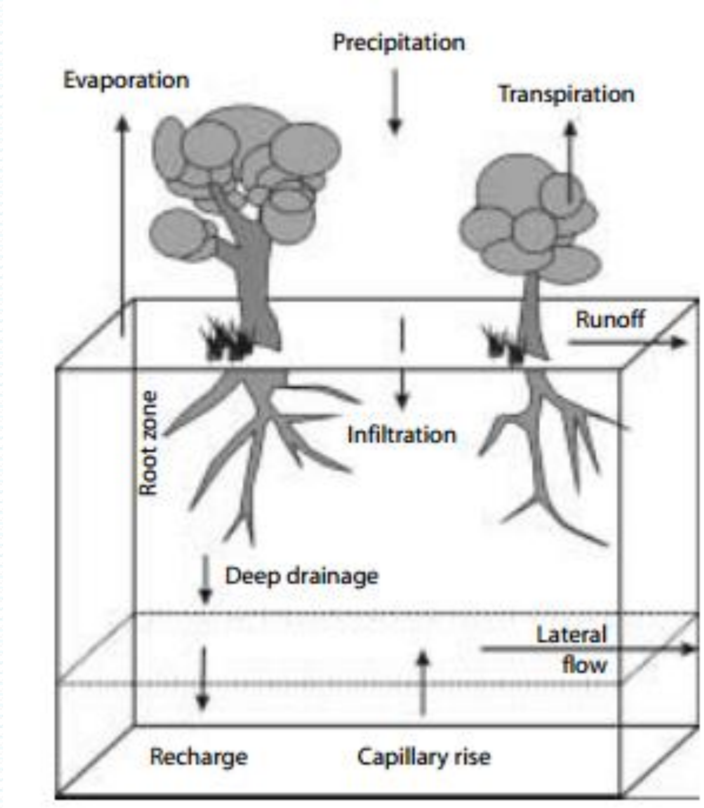


From: The little Prince

Often the traits that we can measure are not the most informative traits

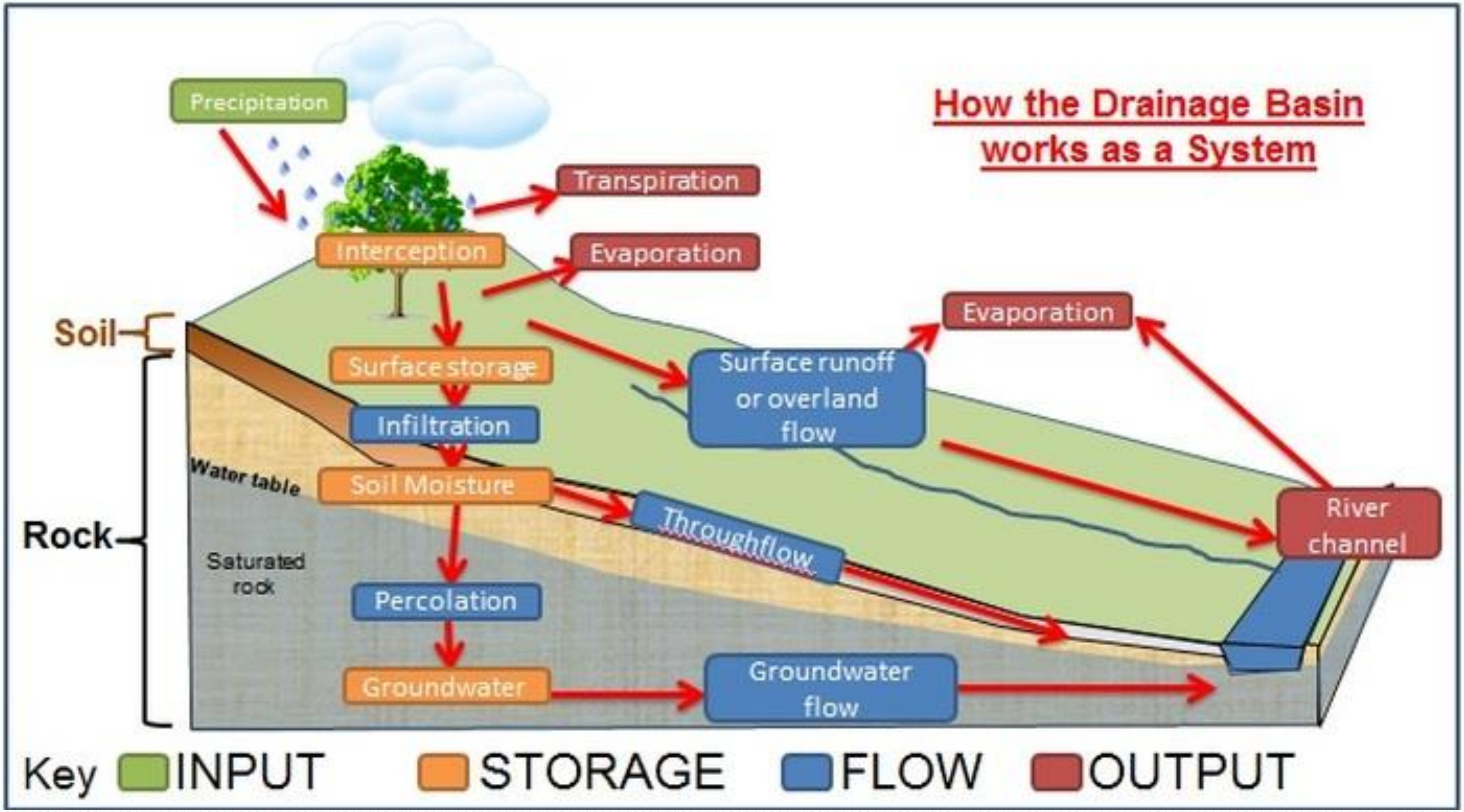
5.2. Conceptual modelling



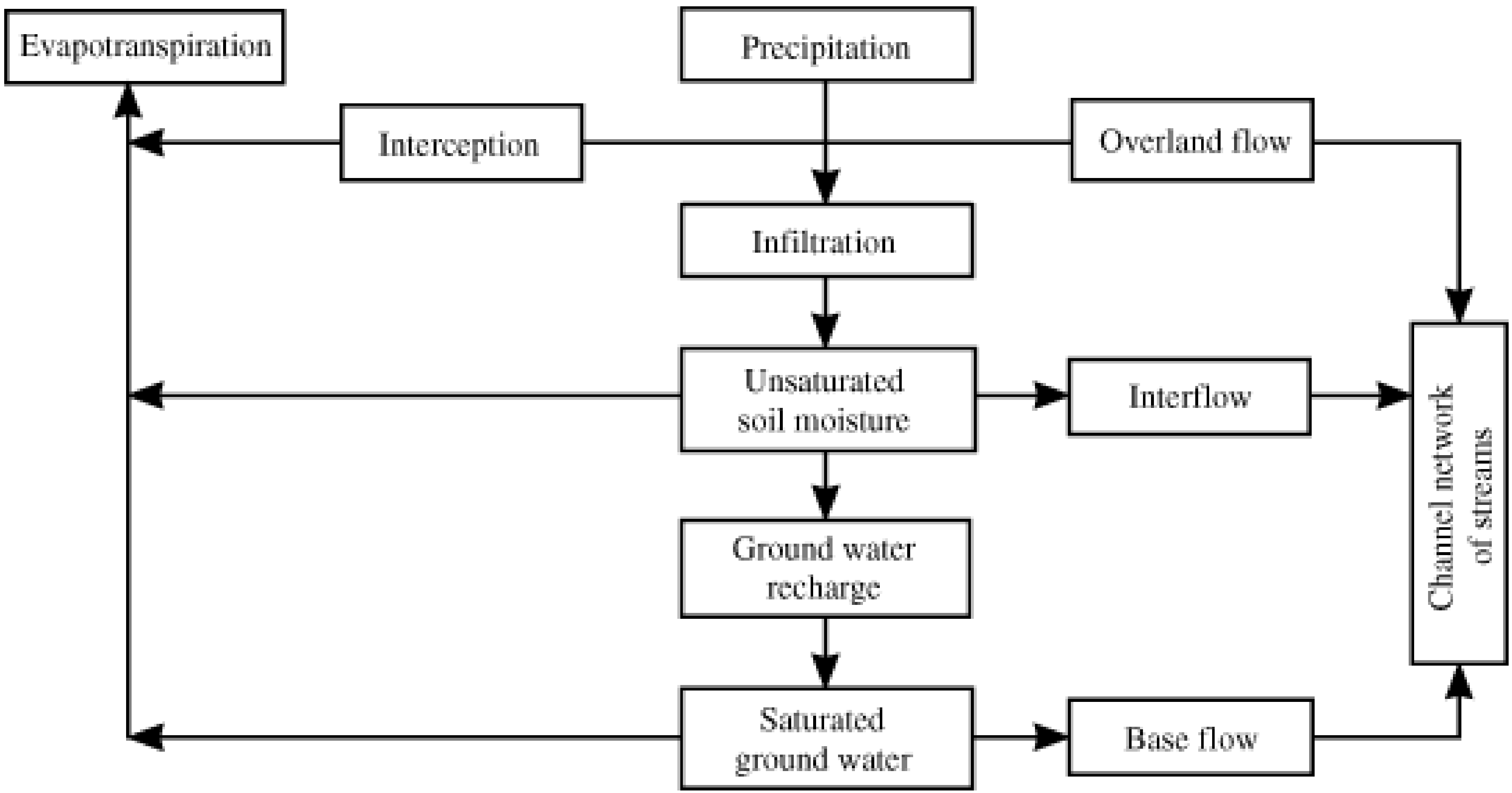


water balance concept

The output of the Conceptual model is a flowchart or diagram



The output of the Conceptual model is a flowchart or diagram

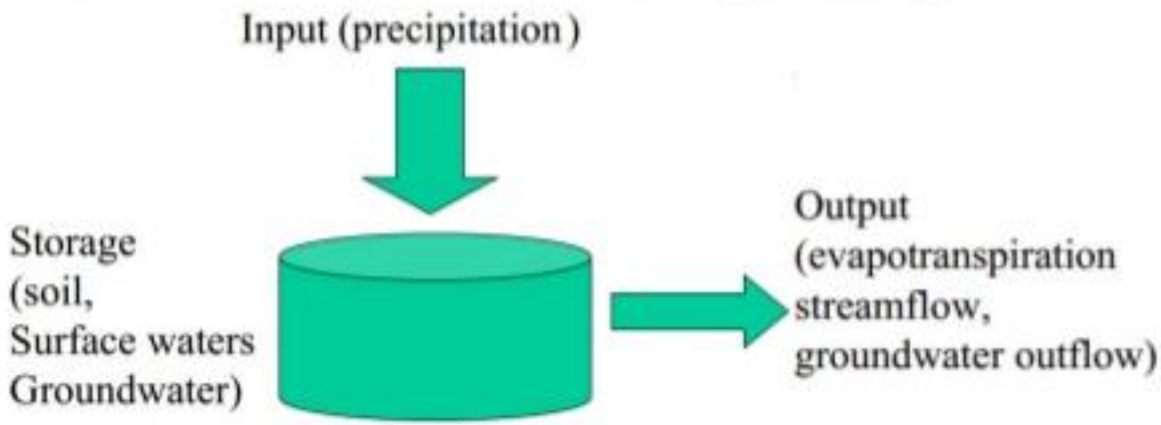


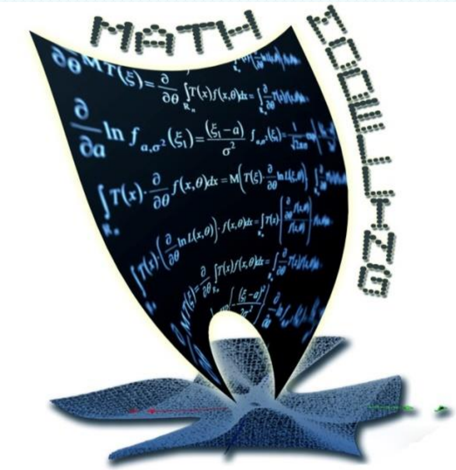
Schematic diagram of the Hydrologic Cycle (after Domenico & Schwartz, 1990)

5.2.2. Numerical modelling

Numerical modelling is composed of variables and a mathematical representation of the relationship between them.

Mass Balance: Input – Output = Change in Storage



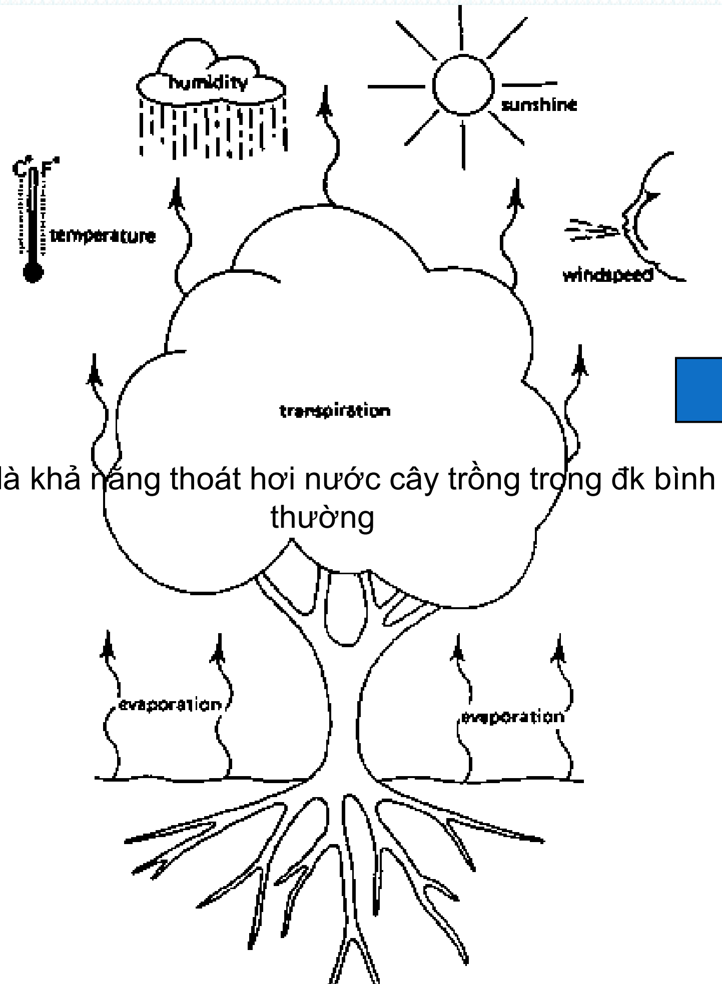


- Water balance equation in its most fundamental form is given by

$$P - Q - E - \Delta S = 0$$

- Where, P=precipitation, E =evaporation, Q = runoff and ΔS = change in storage

5.2.2. Numerical modelling



ET0 là khả năng thoát hơi nước cây trồng trong đk bình thường

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma \frac{900}{T+273} u_2 (e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)}$$

Trong đó :

- ET_0: bốc hơi chuẩn (mm/ngày);
- R_n: bức xạ mặt trời trên bề mặt lá cây trồng (MJ/m^2/ngày);
- G: mật độ hấp thụ nhiệt trong đất (MJ/m^2/ngày);
- T: nhiệt độ bình quân ngày tại chiều cao 2m từ mặt đất (^0C);
- u_2: tốc độ gió tại chiều cao 2m từ mặt đất (m/s);
- e_s: áp suất hơi nước bảo hòa (kPa);
- e_a: áp suất hơi nước thực tế (kPa);
- Δ: độ dốc của áp suất hơi nước trên đường cong quan hệ nhiệt độ (kPa/ ^0C);
- γ: hằng số ẩm (kPa/ ^0C).

$$CWR = Et_c = ET_0 \times k_c \text{ (mm/thời đoạn)}$$

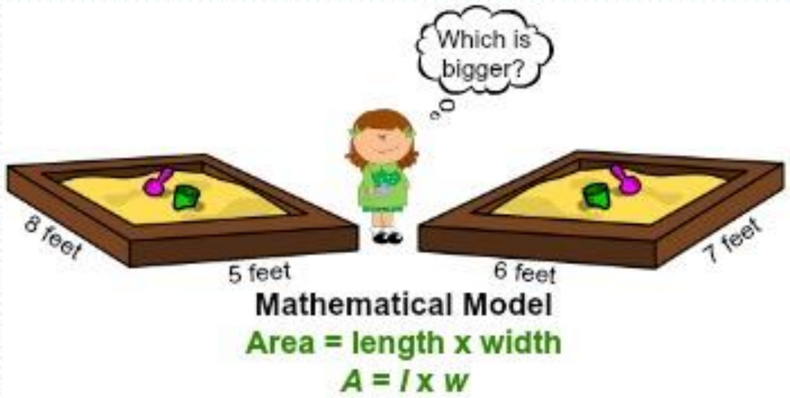
Trong đó:

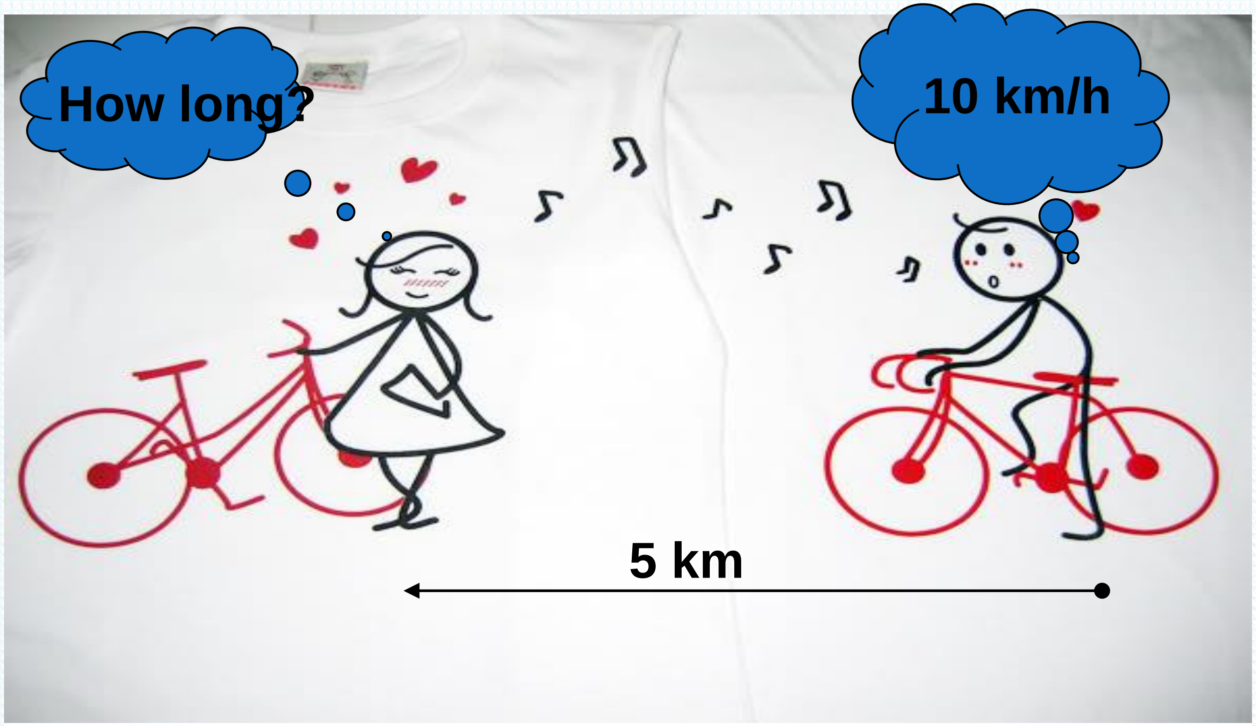
- ET_0: là lượng bốc hơi chuẩn và phụ thuộc hoàn toàn vào các yếu tố khí tượng.
- k_c: là hệ số sinh lý của cây trồng tại thời đoạn tính toán. Hệ số này phụ thuộc vào đặc trưng cây trồng

5.2.2. Numerical modelling

- Forces us to formulate concrete ideas and assumptions in an unambiguous way
- Mathematics is a concise language
- One equation says more than 1000 words
- Mathematics is a universal language
- Same mathematical techniques can be applied over a range of scales
- Mathematics is an old but still trendy language
- The rich toolbox created by mathematicians over centuries is available at our disposal
- Mathematics is the language that computers understand best

5.2.2. Numerical modelling





s: distance

v: speed

$t = s/v$

$t = ?$

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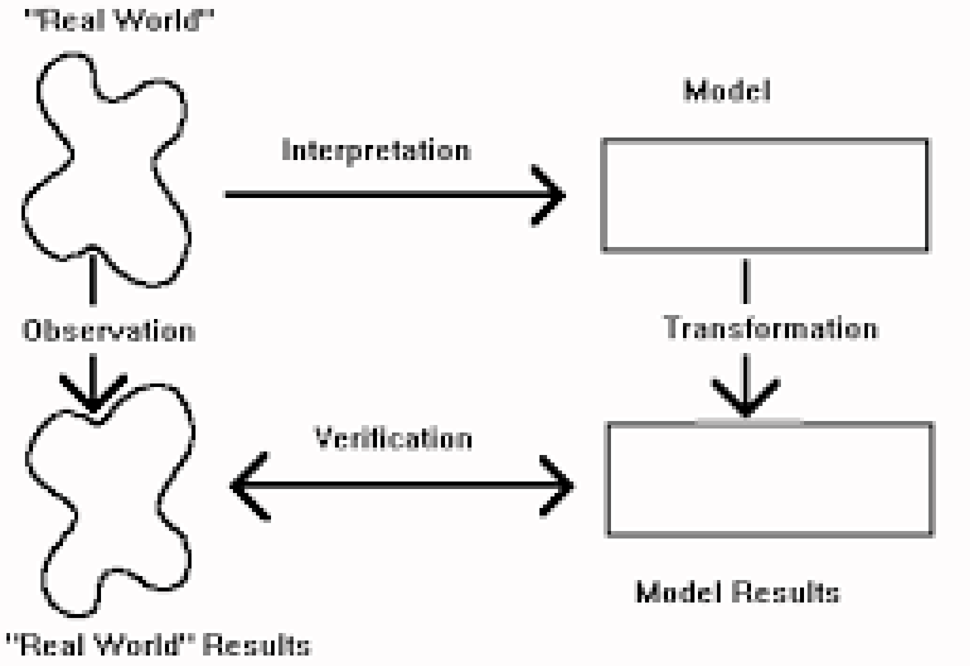
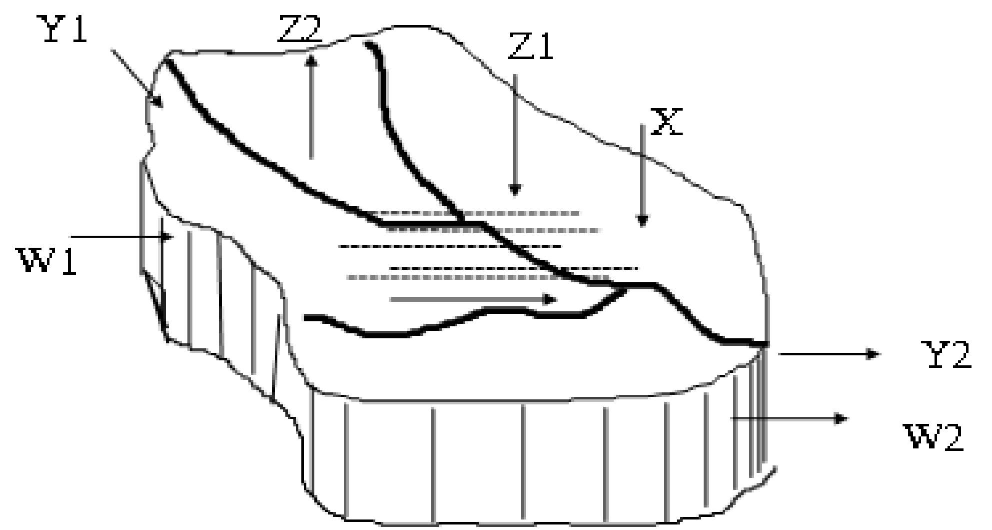
5.2.2. Numerical modelling





$$(X + Z_1 + Y_1 + W_1) - (Z_2 + Y_2 + W_2) = |U_2 - U_1| = \pm\Delta U$$

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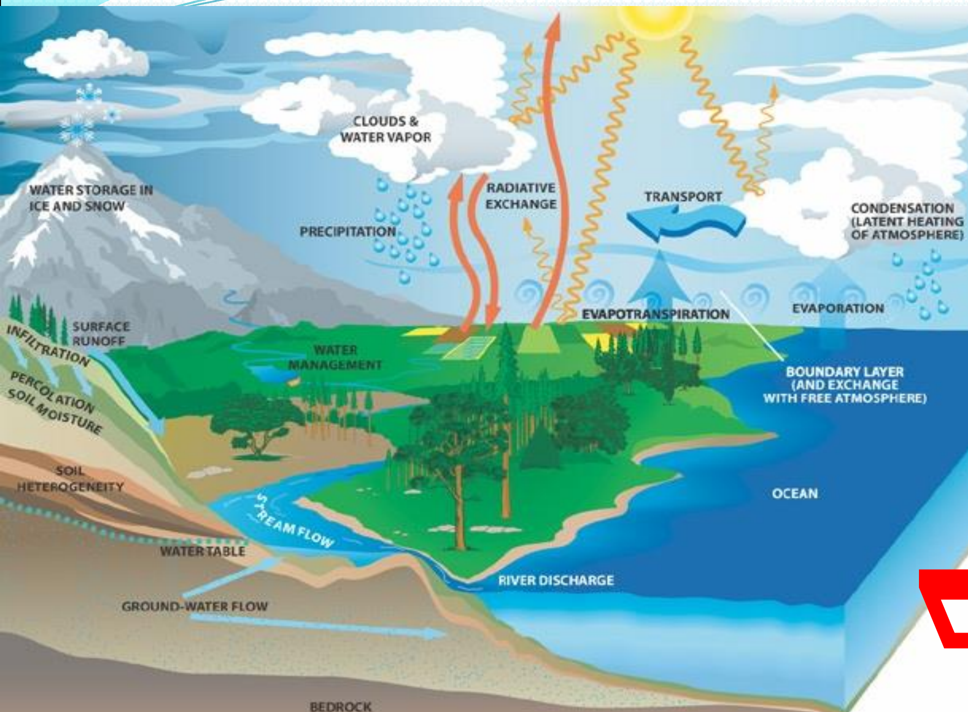

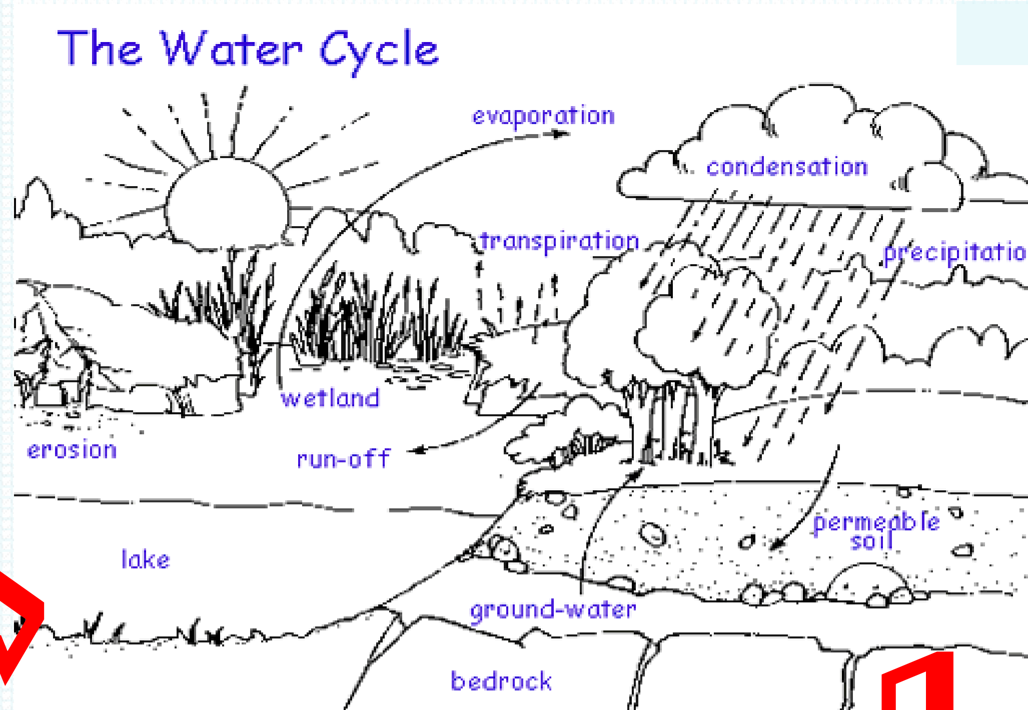

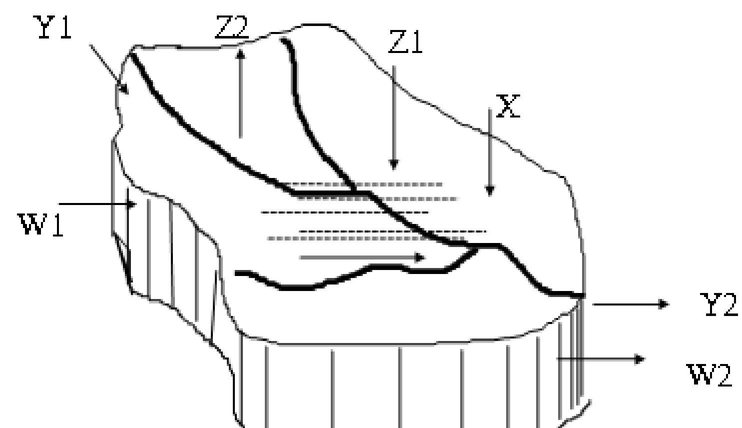

$$(X + Z_1 + Y_1 + W_1) - (Z_2 + Y_2 + W_2) = |U_2 - U_1| = \pm \Delta U$$

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Mathematical modelling classification

Empirical	><	Mechanistic
Deterministic	><	Stochastic
Systems	><	Molecular model
Static	><	Dynamic
Linear	><	Non-linear
Discrete	><	Continuous

Classifying them into broad categories can tell you much about their purpose & scope and often require different mathematical techniques

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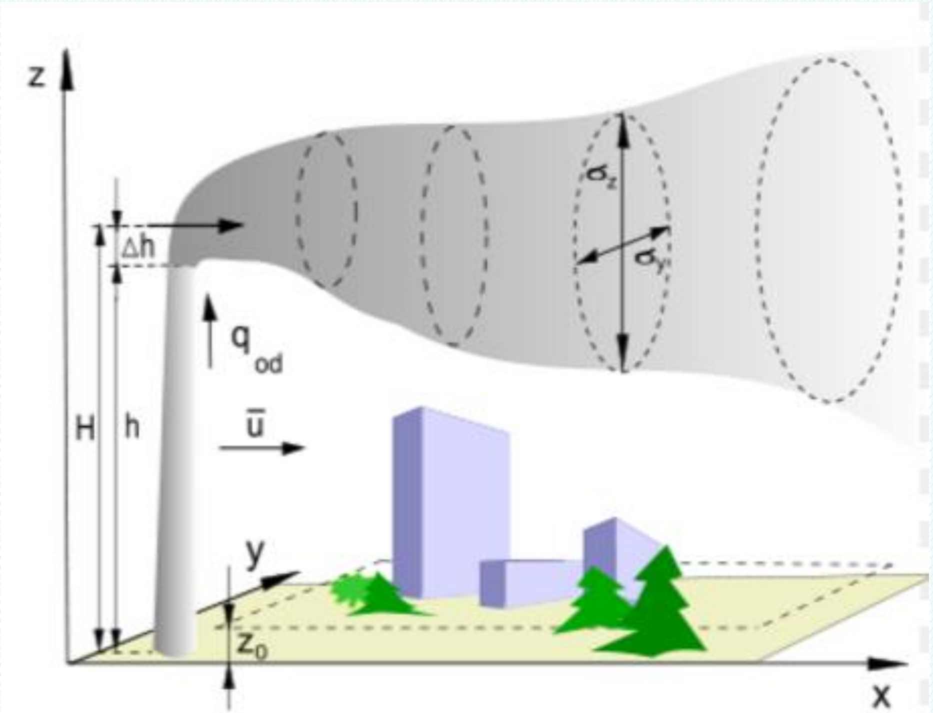
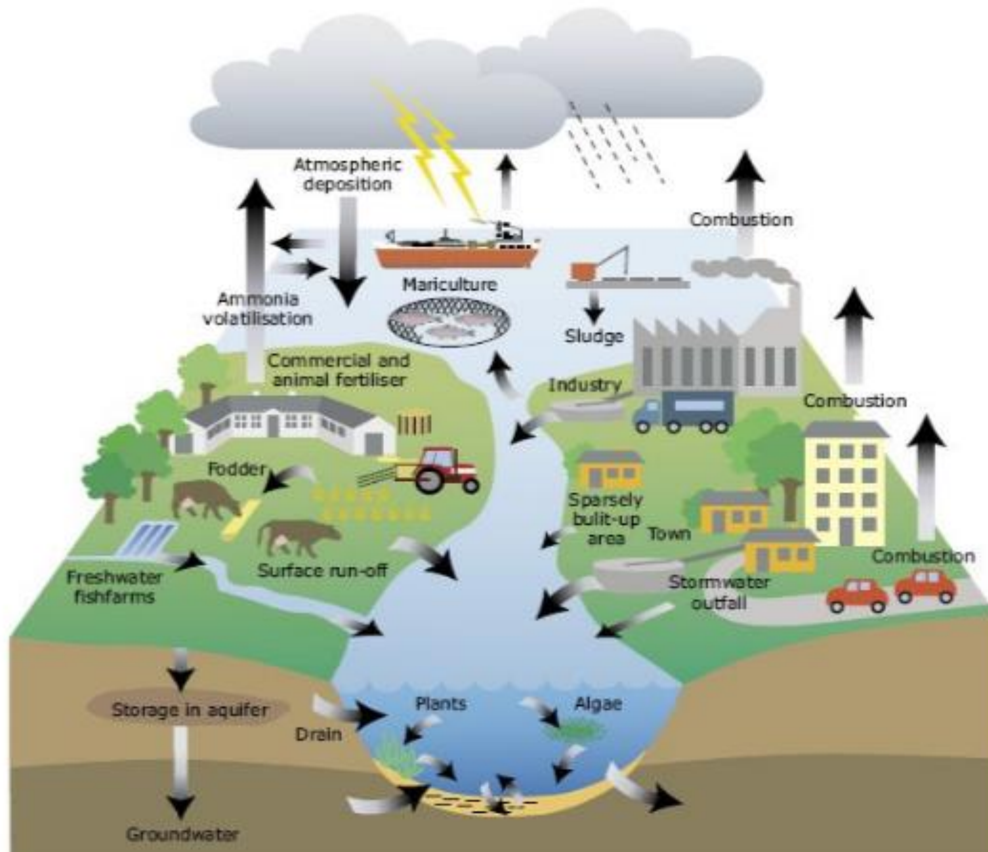
6. Environmental modelling

Part 1



$$C_{od,\,xyz} = \frac{q_{od}}{2\pi \bar{u}\,\sigma_y \sigma_z} \exp\left[-\frac{y^2}{2\sigma_y^2}\right]\left\{\exp\left[-\frac{(z-H)^2}{2\sigma_z^2}\right] + \exp\left[-\frac{(z+H)^2}{2\sigma_z^2}\right]\right\}$$

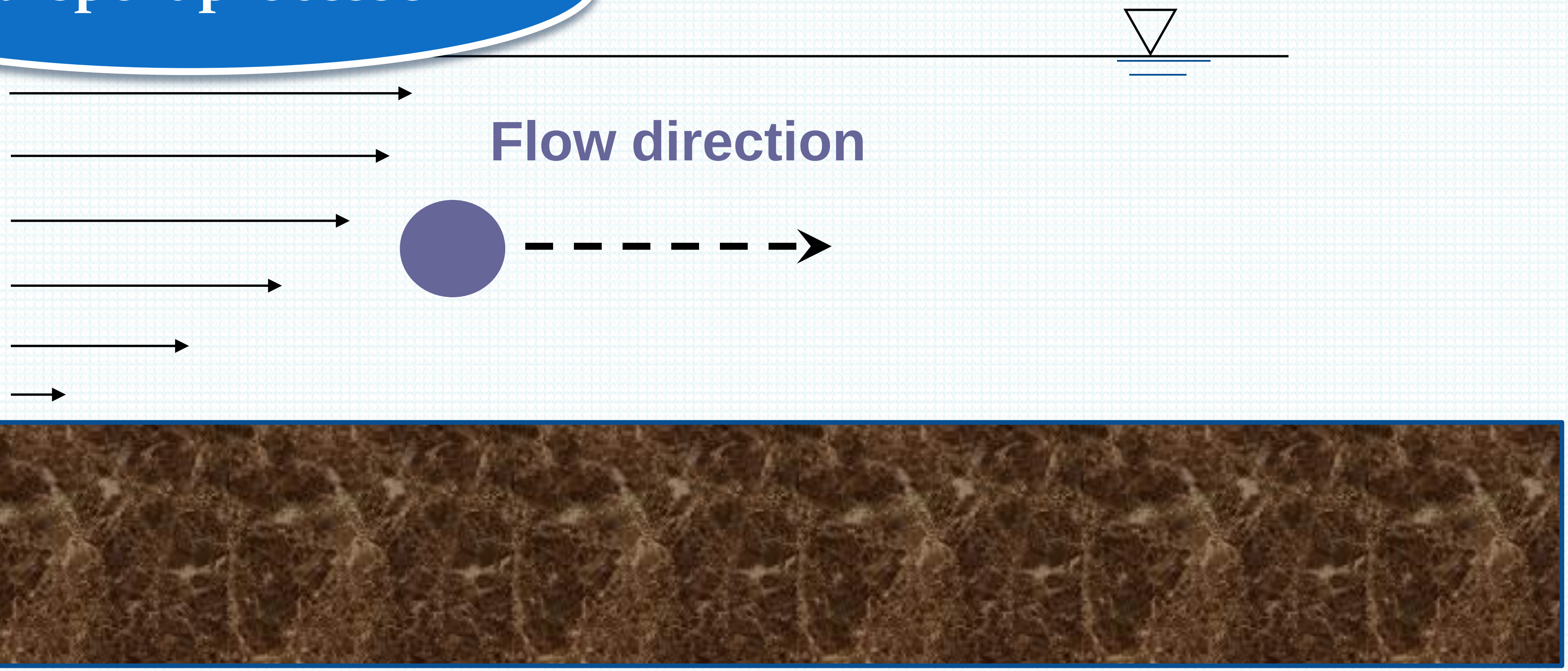


6. Environmental modelling

Part 1

Water quality modelling

Transport processer



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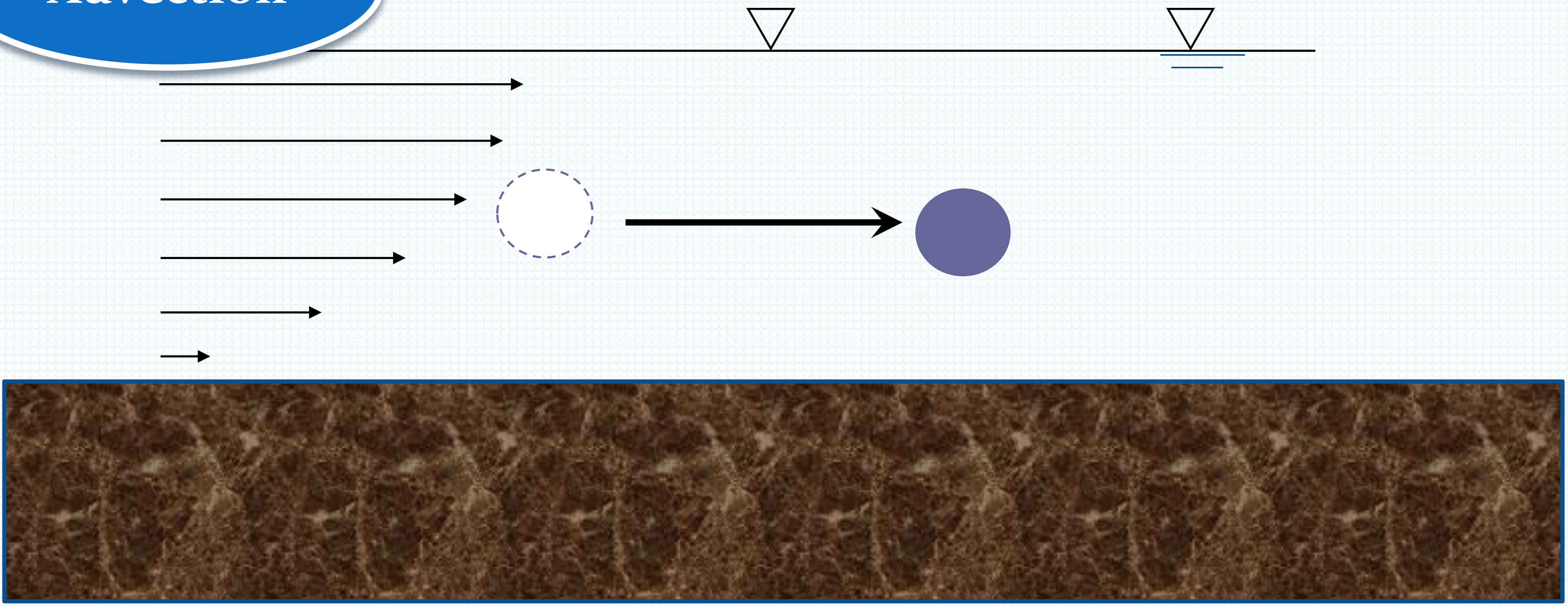
6. Environmental modelling

Water quality modelling

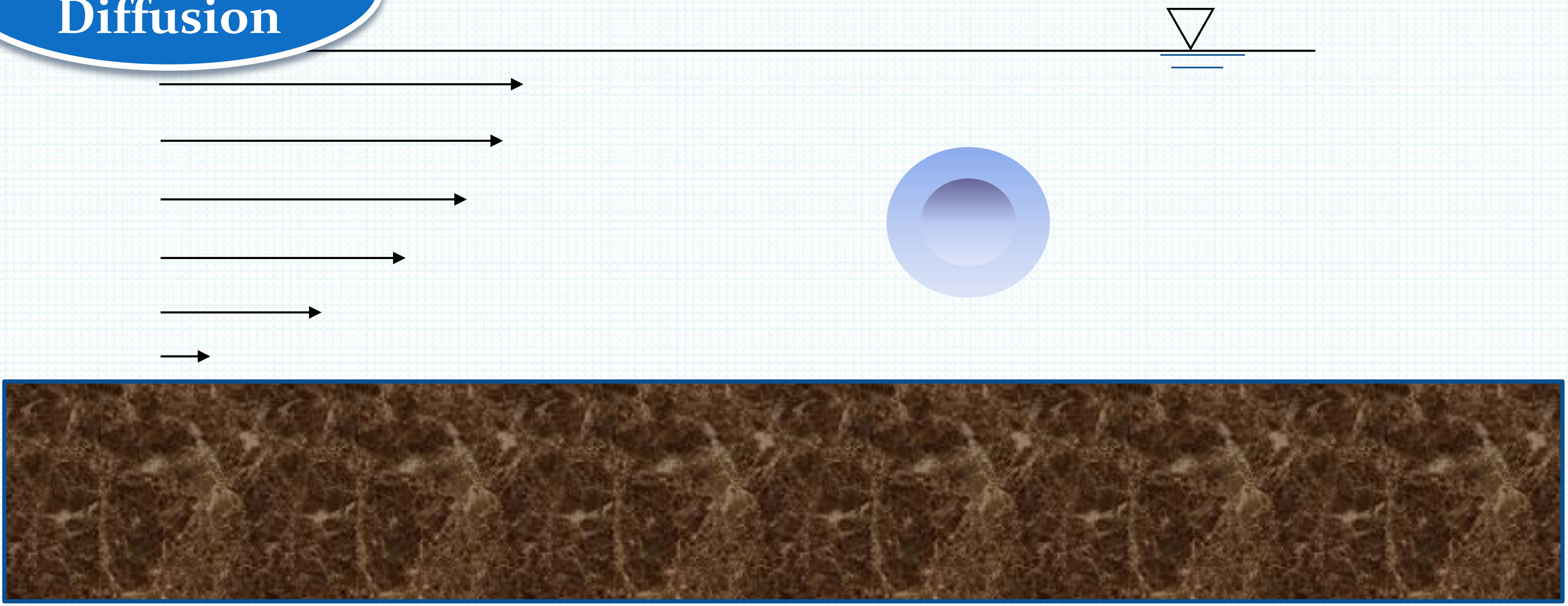
Advection



6. Environmental modelling

Water quality modelling

Molecular Diffusion



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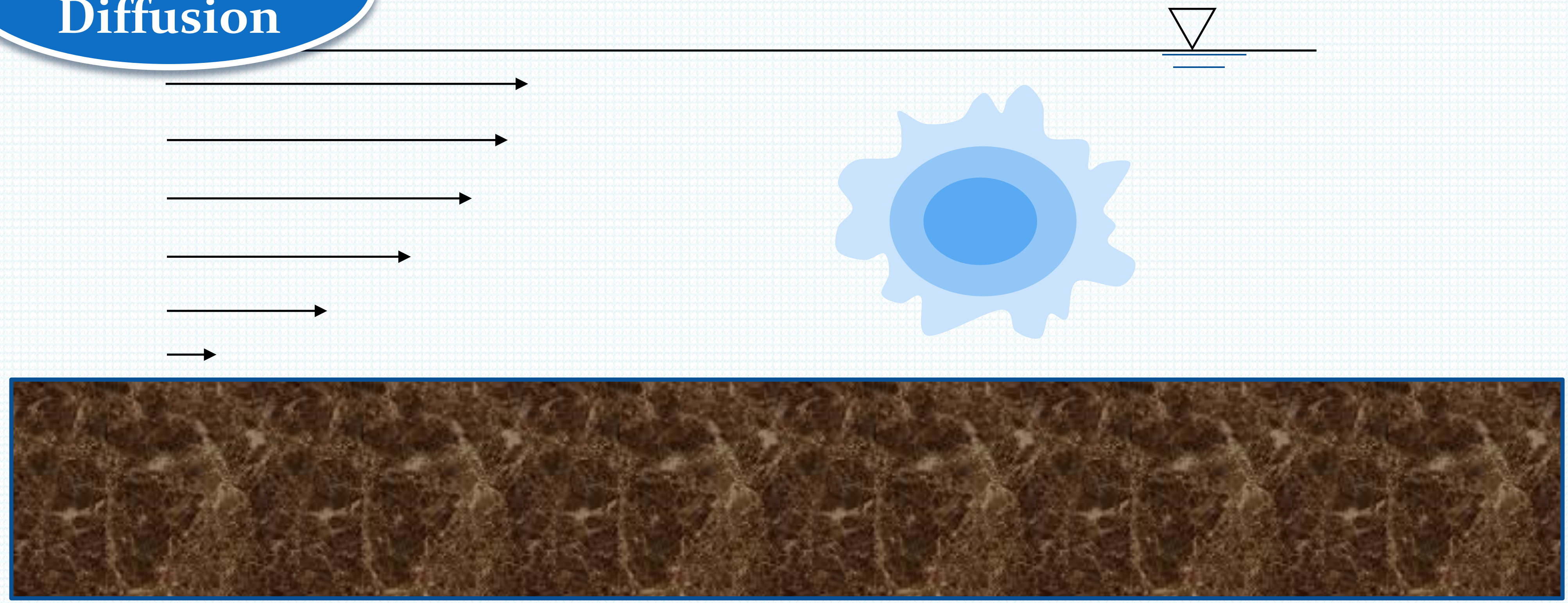
6. Environmental modelling

Water quality modelling

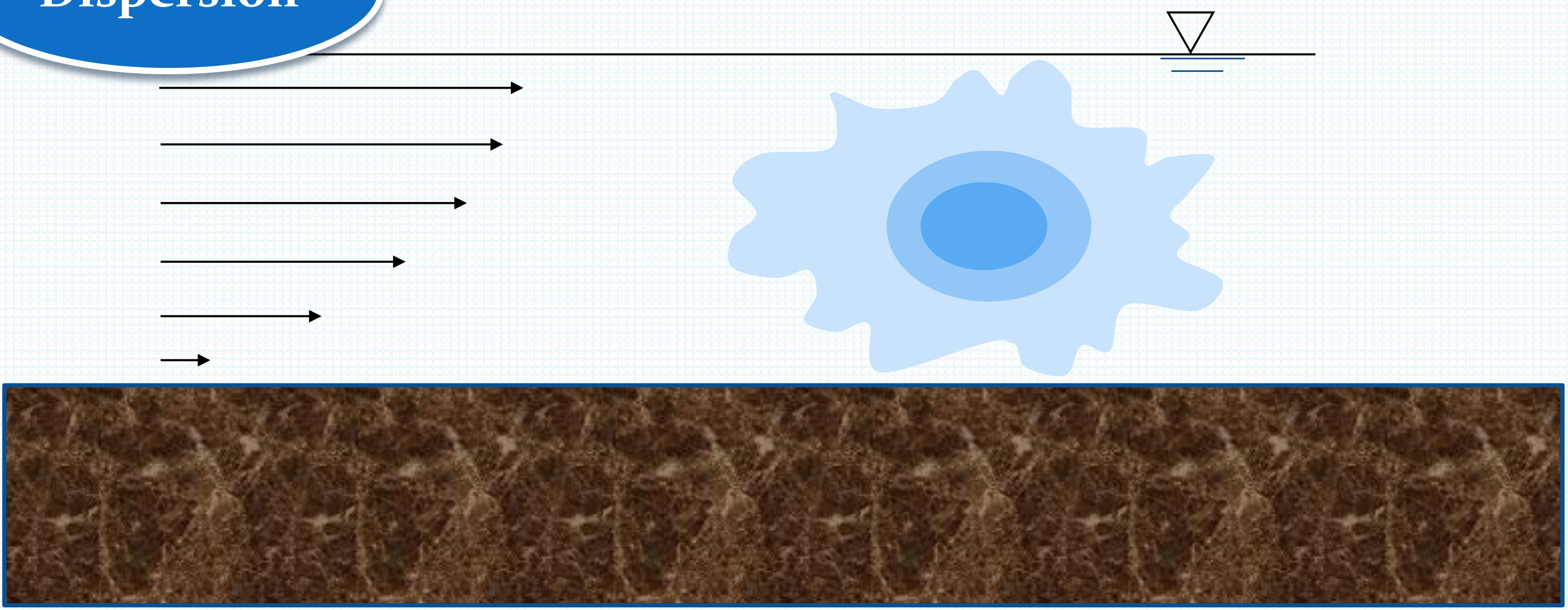
Turbulent Diffusion



6. Environmental modelling

Water quality modelling

Dispersion



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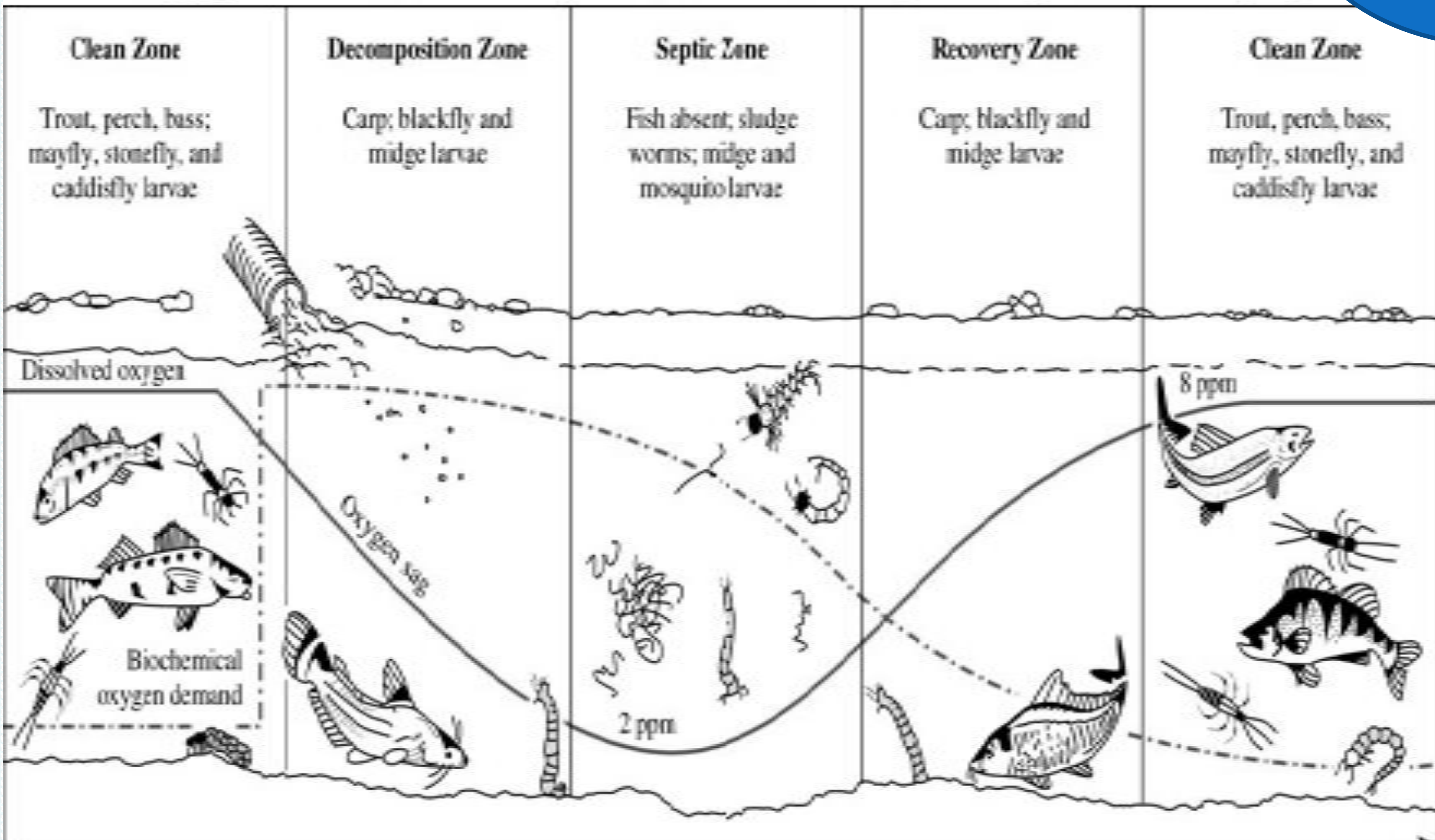
6. Environmental modelling

Oxygen depletion in streams



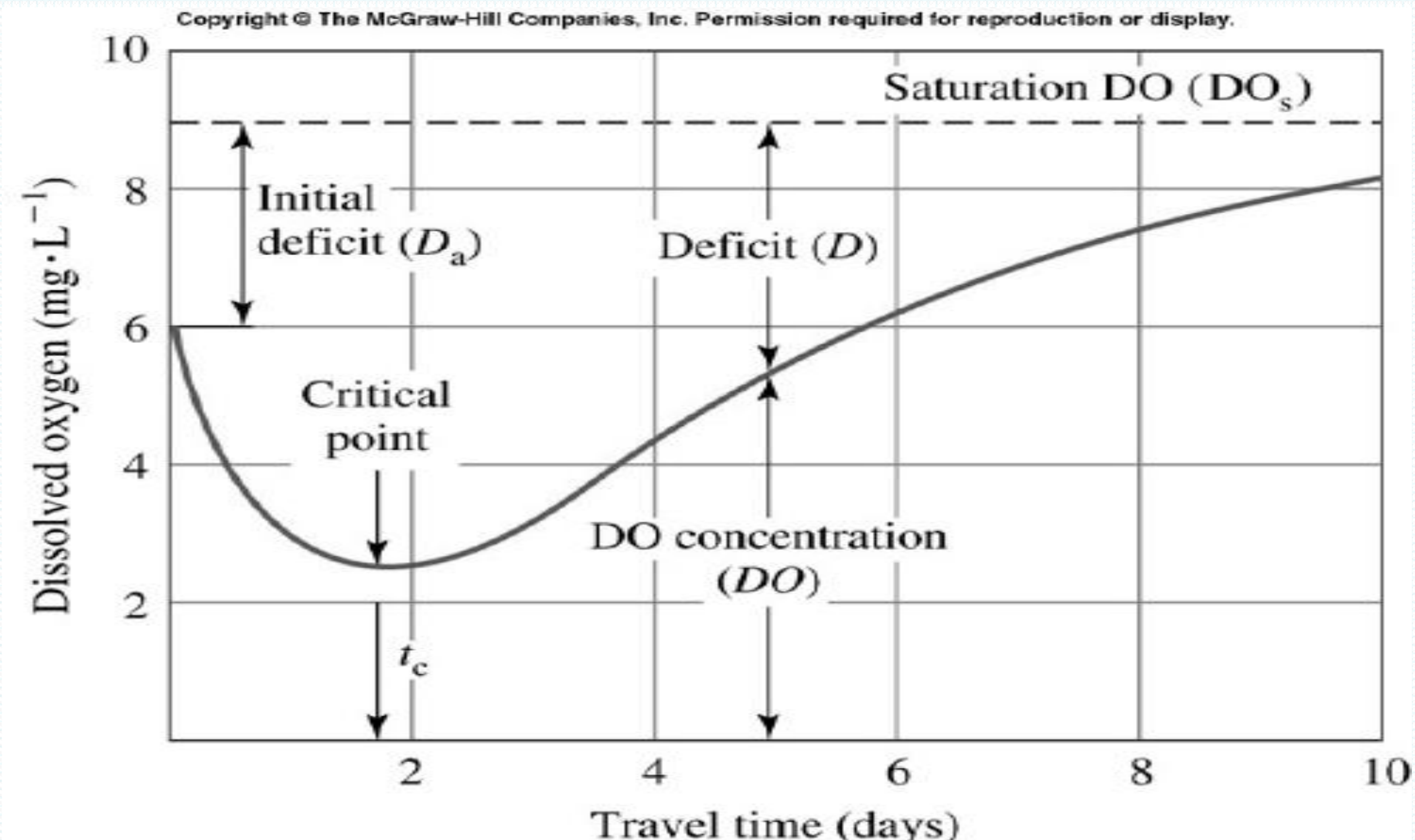
- Factors Affecting Amount of DO Available in Rivers
- Oxygen demanding wastes affect available DO
- Tributaries bring their own oxygen supply
- Photosynthesis adds DO during the day but the same plants remove oxygen at night
- Respiration of organisms living in water as well as in sediments remove oxygen
- In the summer rising temperatures reduce solubility of oxygen
- In the winter oxygen solubility increases, but ice may form blocking access to new atmospheric oxygen

Modeling DO in a River

- To model all the effects and their interaction is a difficult task
- The simplest model focuses on two processes:
    - ➢ The removal of oxygen by microorganisms during biodegradation (de-oxygenation)
    - ➢ The replenishment of oxygen at the interface between the river and the atmosphere (re-aeration)

6. Environmental modelling

DO sag definitions



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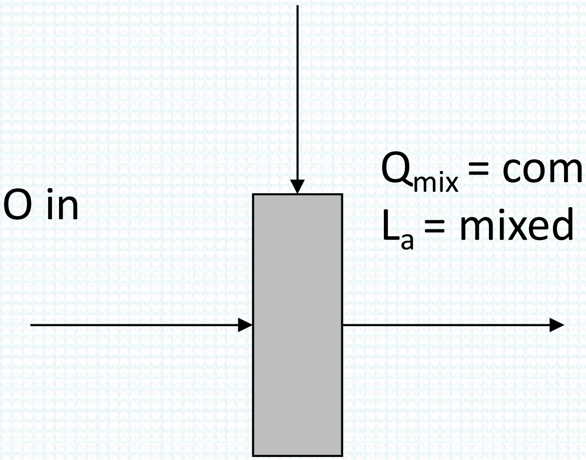
Steps in Developing the DO Sag Curve

1. Determine the initial conditions
2. Determine the de-oxygenation rate from BOD test and stream geometry
3. Determine the re-aeration rate from stream geometry
4. Calculate the DO deficit as a function of time
5. Calculate the time and deficit at the critical point (worst conditions

Mass Balance for Initial Mixing

Q_w = waste flow (m^3/s) DO_w = DO in waste (mg/L) L_w = BOD in waste (mg/L)

Q_r = river flow (m^3/s) DO_r = DO in river (mg/L) L_r = BOD in river (mg/L)



Q_{mix} = combined flow (m^3/s) DO = mixed DO (mg/L) L_a = mixed BOD (mg/L)

1. Determine Initial Conditions

Initial dissolved oxygen concentration:

$$DO = \frac{Q_w DO_w + Q_r DO_r}{Q_w + Q_r}$$

Initial DO deficit:

$$D_a = DO_s - DO$$

where:

D_a=initial DO deficit (mg/L)

DO_s=saturation DO conc.(mg/L)

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1. Determine Initial Conditions

Therefore, the initial deficit after mixing is

$$D_a = DO_s - \frac{Q_w DO_w + Q_r DO_r}{Q_{mix}}$$

where D_a is the initial deficit (mg/L)
Note: DO_s is a function of temperature, atmospheric pressure, and salinity. Values of DO_s are found in tables.

1. Determine Initial Conditions

Solubility of Oxygen in Water (DOs = DO saturation)

DO_s is a function of temperature, atmospheric pressure and salinity

	Chloride concentration in water (mg/L)			
Temperature (°C)	0	5000	10,000	15,000
0	14.62	13.73	12.89	12.10
5	12.77	12.02	11.32	10.66
10	11.29	10.66	10.06	9.49
15	10.08	9.54	9.03	8.54
20	9.09	8.62	8.17	7.75
25	8.26	7.85	7.46	7.08
30	7.56	7.19	6.85	6.51

Source: Thomann and Mueller (1987).

1. Determine Initial Conditions

Initial ultimate BOD concentration: If, the BOD data for the waste or river are in terms of BOD_5, calculate L for each

$$L = \frac{BOD_t}{1 - e^{-kt}}$$

Therefore, initial *ultimate* BOD concentration

$$L_a = \frac{Q_w L_w + Q_r L_r}{Q_w + Q_r}$$

2. Determine de-oxygenation rate

rate of de-oxygenation = $k_d L_t$

where: k_d = de-oxygenation rate coefficient (day^{-1})

L_t = ultimate BOD remaining at time (of travel down-stream) t

If k_d (stream) = k (BOD test) and $L_t = L_0 e^{-k_d t}$

$$\text{rate of de-oxygentation} = k_d L_0 e^{-k_d t}$$

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3. Determine Re-aeration Rate

rate of re-aeration = k_r D

k_r = re-aeration constant (time $^{-1}$)
D = dissolved oxygen deficit (DO_s-DO)
DO_s = saturated value of oxygen
DO = actual dissolved oxygen at a given location downstream

3. Determine Re-aeration Rate

$$k_r = \frac{3.9u^{1/2}}{h^{3/2}}$$

where k_r = re-aeration coefficient @ 20ºC (day^{-1})

u = average stream velocity (m/s)

h = average stream depth (m)

Correct rate coefficient for stream temperature

$$k_r = k_{r,20}\Theta^{T-20}$$

where Θ = 1.024

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4. DO deficit as a function of time

DO as function of time (Streeter-Phelps equation or oxygen sag curve)

Rate of increase of DO deficit = rate of deoxygenation – rate of reaeration

$$\frac{dD}{dt} = k_d L_t - k_r D$$

Solution is:

$$D_t = \frac{k_d L_o}{k_r - k_d}\left(e^{-k_d t} - e^{-k_r t}\right) + D_a\left(e^{-k_r t}\right)$$

5. Critical time and DO

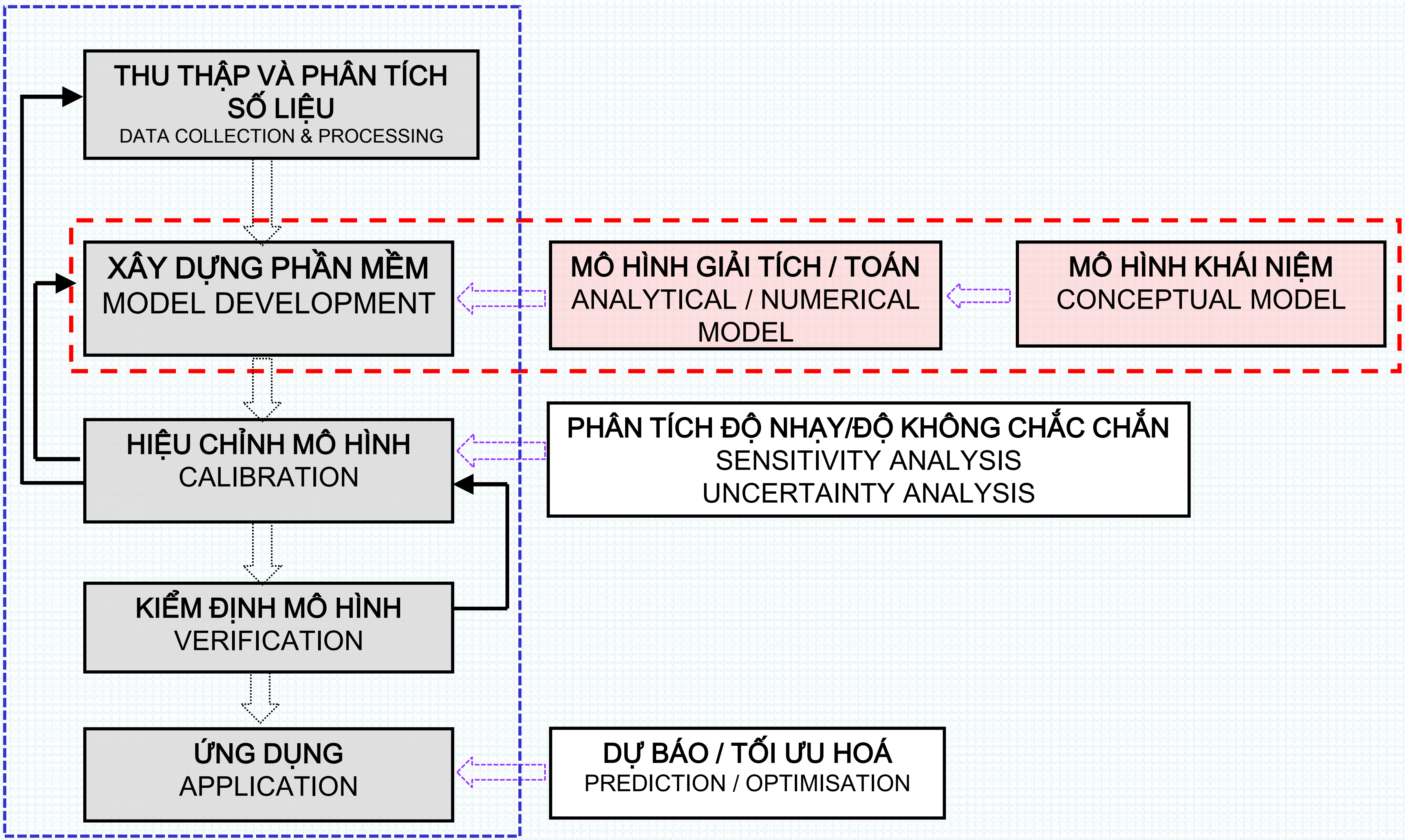
Critical Point = point where steam conditions are at their worst

$$t_c = \frac{1}{k_r - k_d} \ln\left[\frac{k_r}{k_d}\left(1 - D_a \frac{k_r - k_d}{k_d L_a}\right)\right]$$

$$D_c = \frac{k_d L_a}{k_r - k_a}\left(e^{-k_d t_c} - e^{-k_r t_c}\right) + D_a e^{-k_r t_c}$$

D = dissolved oxygen deficit

STEPS FOR MODEL DEVELOPMENT



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STEPS FOR MODEL DEVELOPMENT

Part 2

2.1. data collection) – Input model

- Types of data (Water level, discharge, parameters,…)
- Determine the time step (daily, hourly,…)
- Determine the period of the data series simulate (hours/day/month/year or tens of years,…)
- Determine time to collect/measure data
- Assess the level of reliability, assess the possibility of frequently of the collected data.

Remember: The quality of output data is no better than the quality of Input data

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Part 2

2.1. data collection) –
Input model

Data collection for:

(i) Điều kiện ban đầu (initial data);

(ii) Dữ liệu cho điều kiện biên (boundaries data);

(iii) Hiệu chỉnh mô hình (calibration);

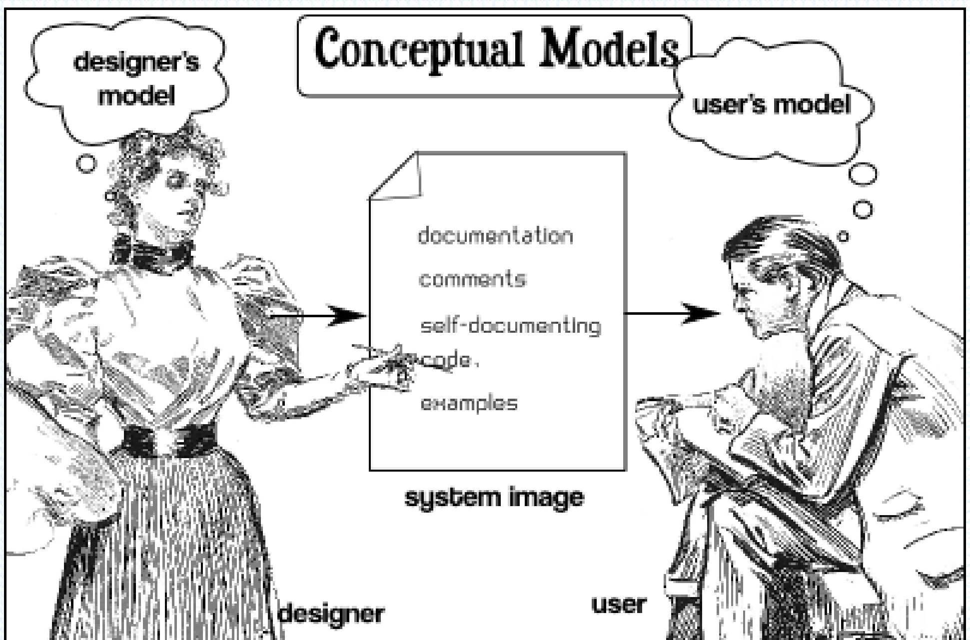
(iv) Kiểm định mô hình (verification).

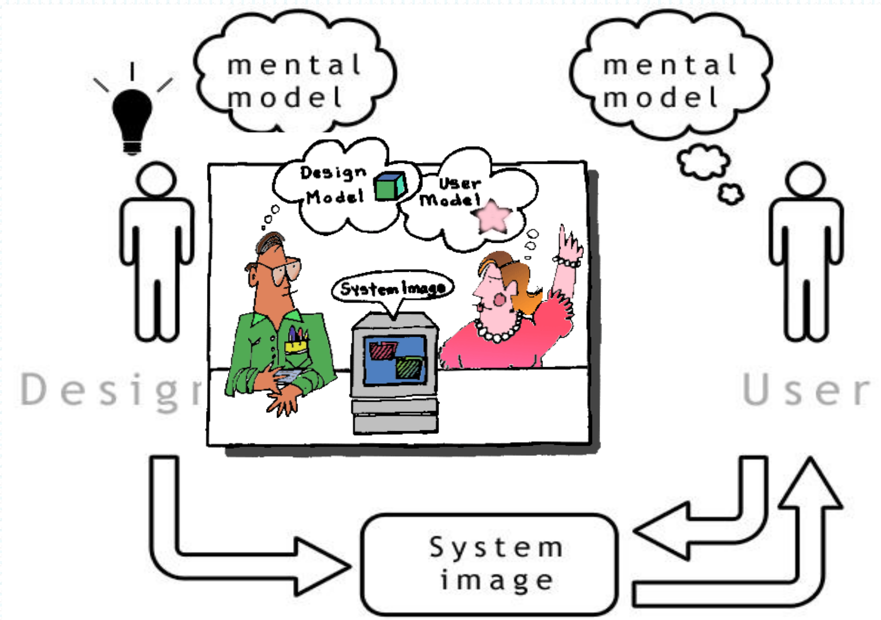
The input data requirements vary according on the type of model

2.2. Model development

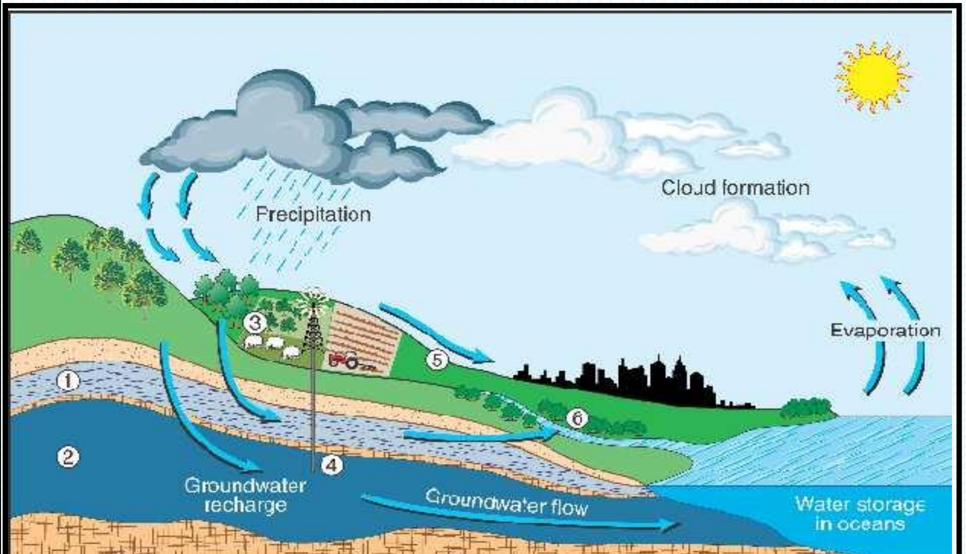
XÂY DỰNG PHẦN MỀM MODEL DEVELOPMENT	⇐	MÔ HÌNH GIẢI TÍCH / TOÁN ANALYTICAL / NUMERICAL MODEL	⇐	MÔ HÌNH KHÁI NIỆM CONCEPTUAL MODEL

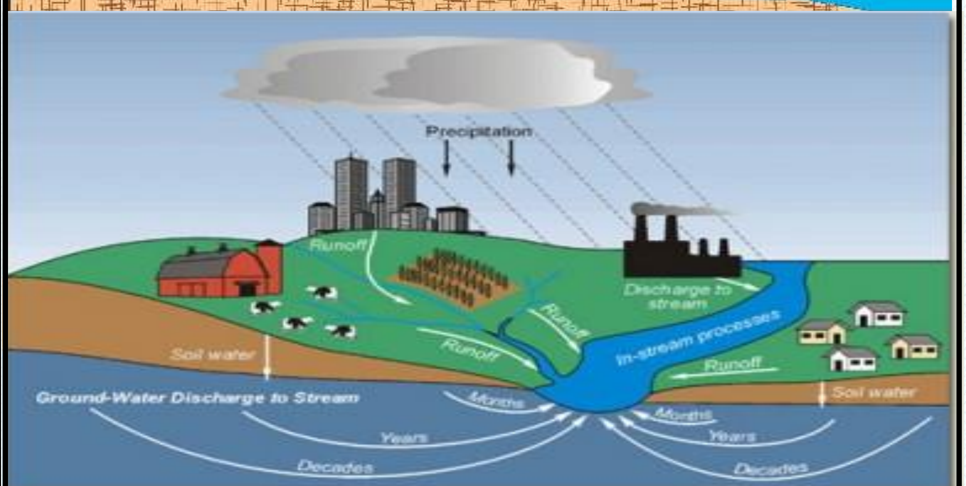
Conceptual modelling

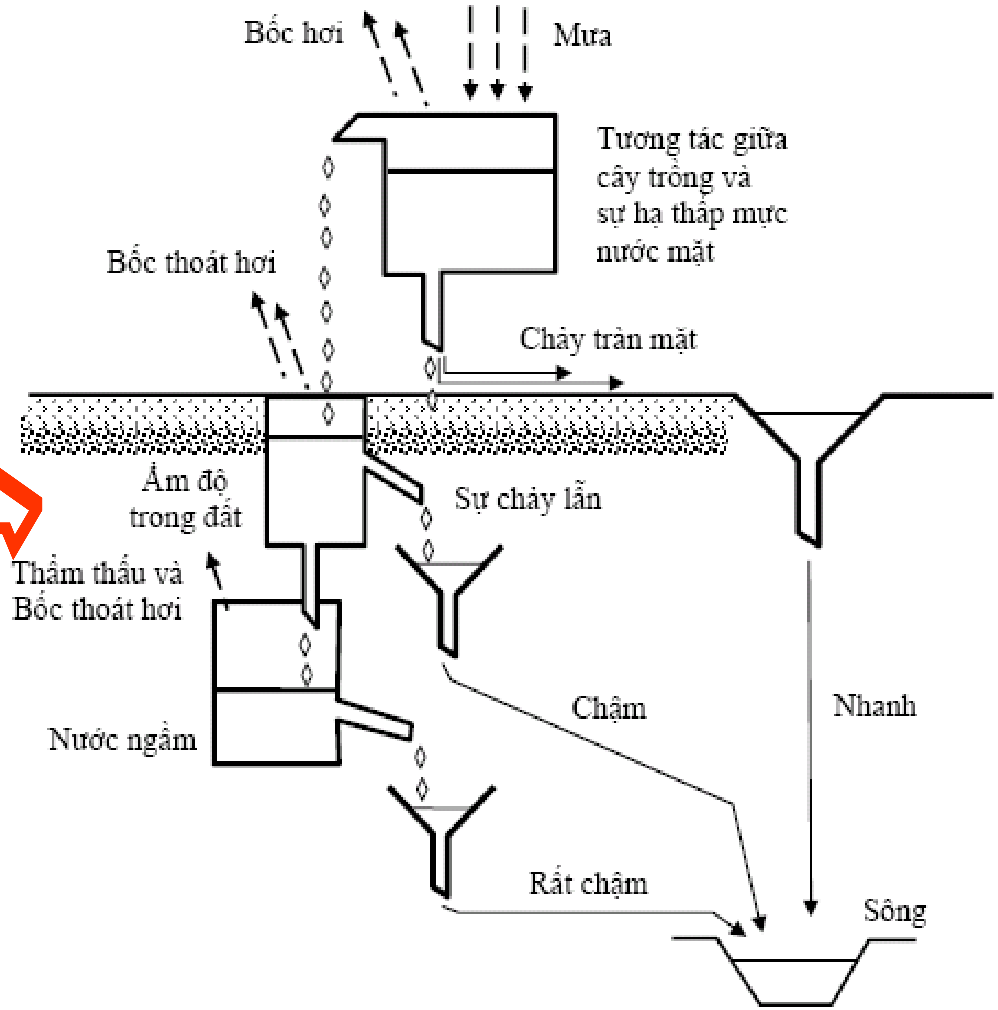




Conceptual modelling example





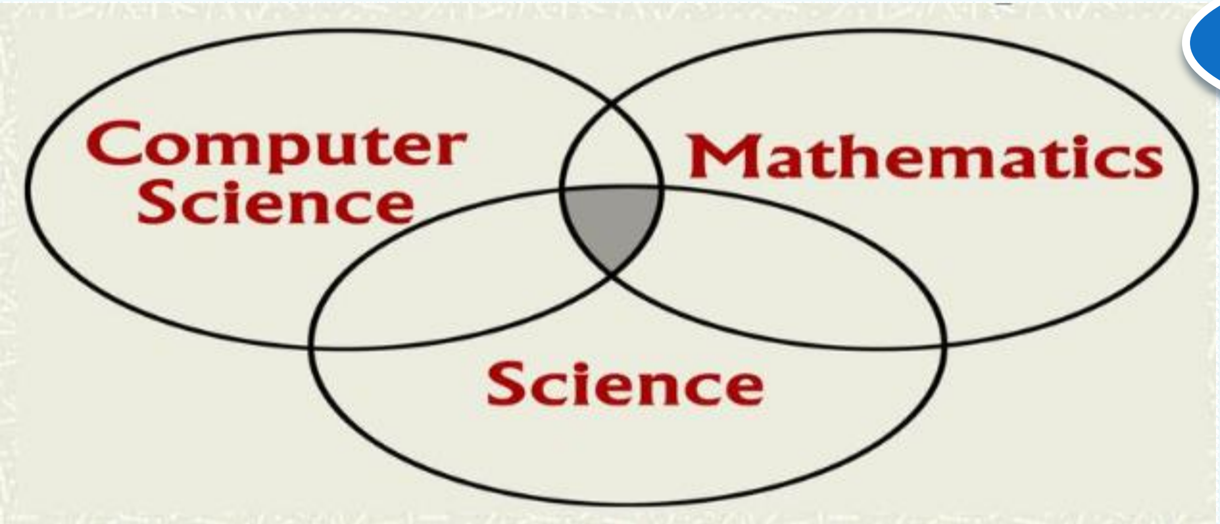


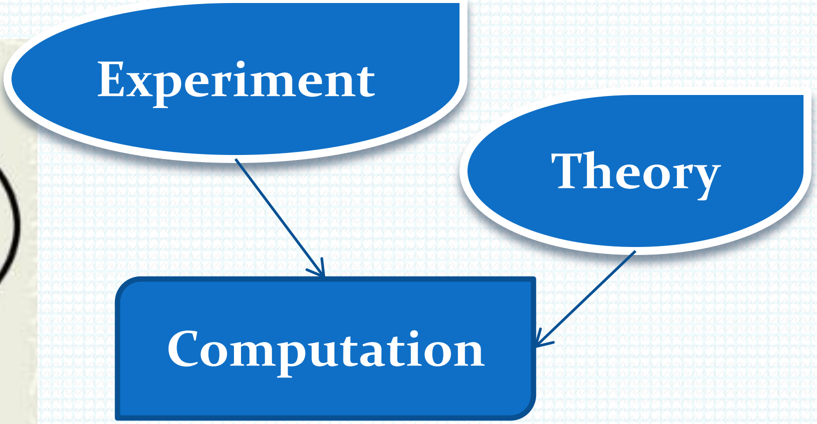
2.3. Numerical modelling

Numerical modelling is composed of variables and a mathematical representation of the relationship between them

2.4. Mathematical modelling

Mathematical modelling involves teamwork





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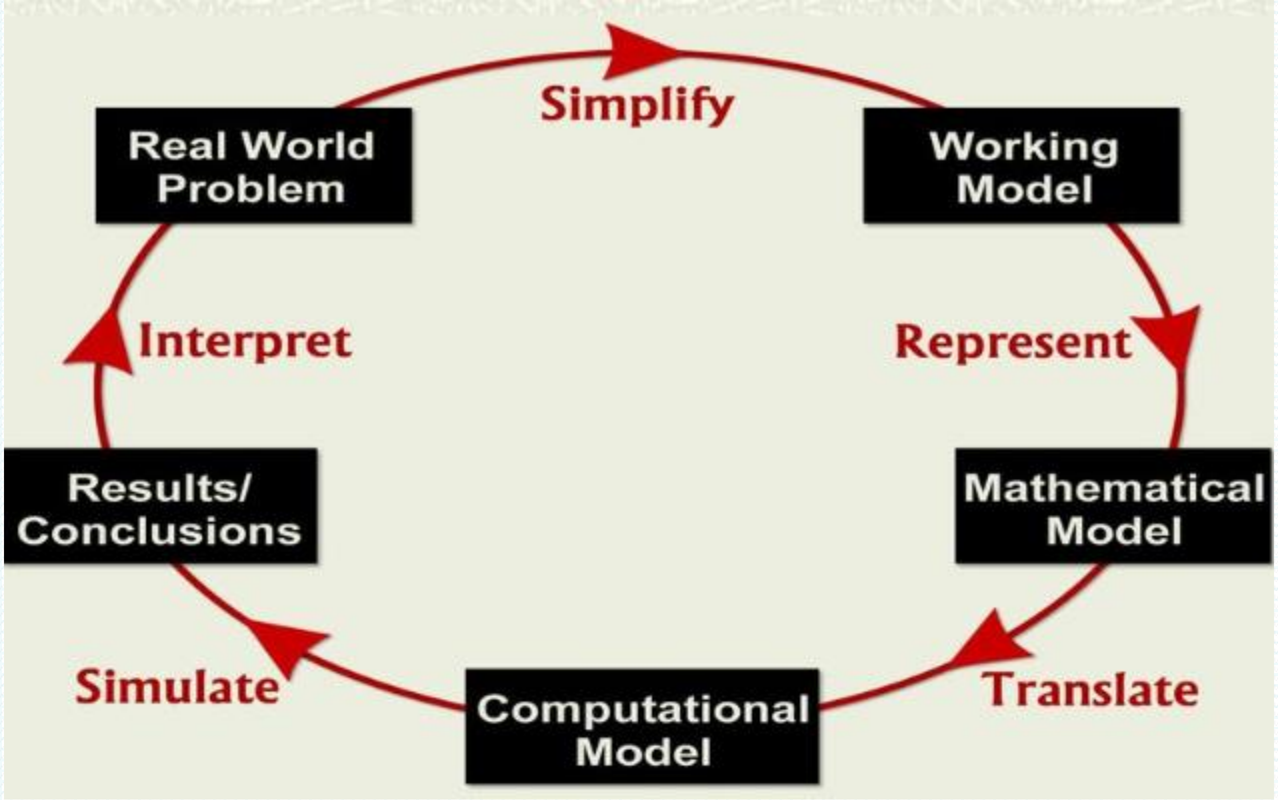
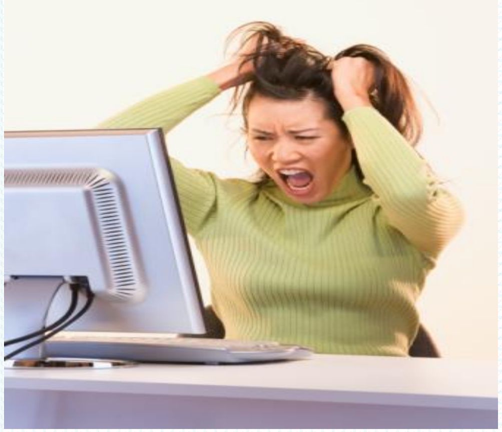
2.4. Mathematical modelling

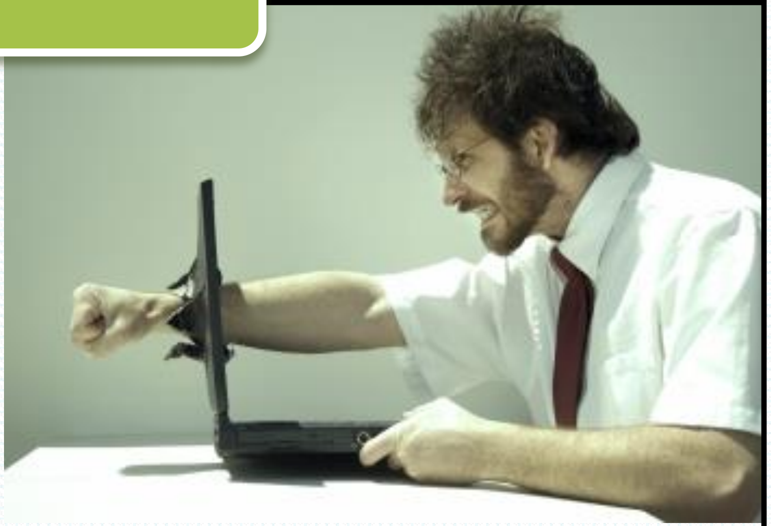


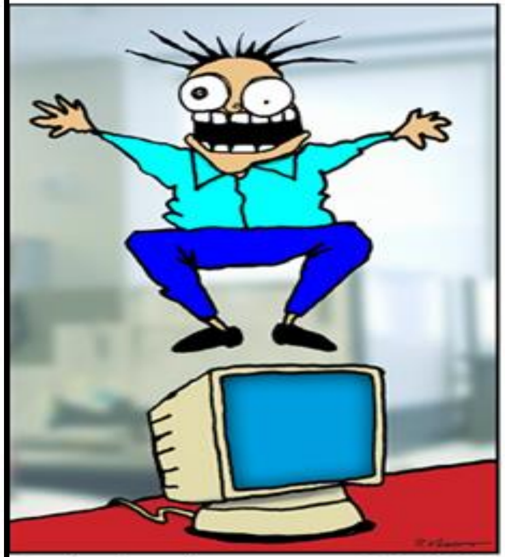
Mathematical modelling process

MODEL CALIBRATION

Calibration

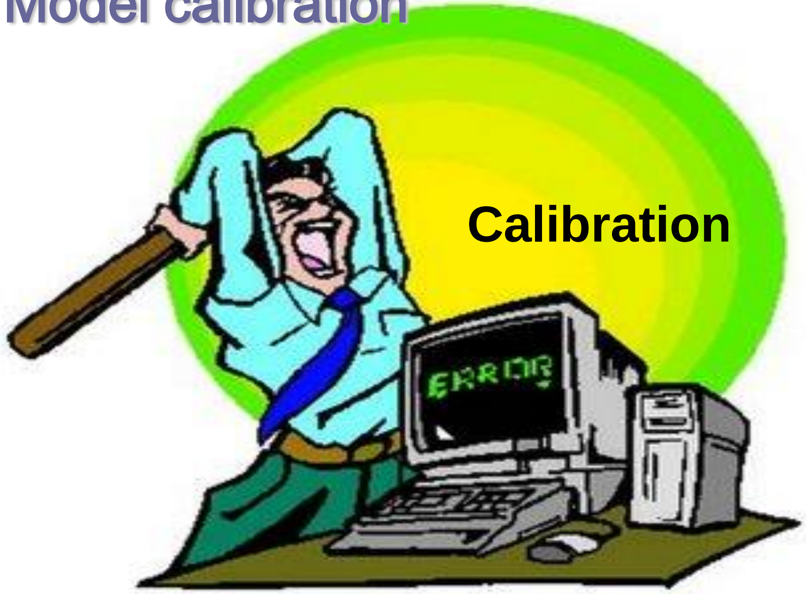






CALIBRATION

Model calibration



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MODEL CALIBRATION

☹ **Adjust the set of parameters until the data output from the model (simulated data) goodness of fit with the (observed data).**

☹ In case the model correction is not satisfied ➔ Check the collected data and Conceptual model.

☹ The sensitivity of the parameters must be examined in this step.

What are the results from model calibration?

MODEL VERIFICATION

- ✓ Use another set of data to check the value of the parameter set in the model.
- ✓ If cross-validation fails, re-calibrate the model's parameter set until satisfied (return to calibration step).

MODEL VERIFICATION

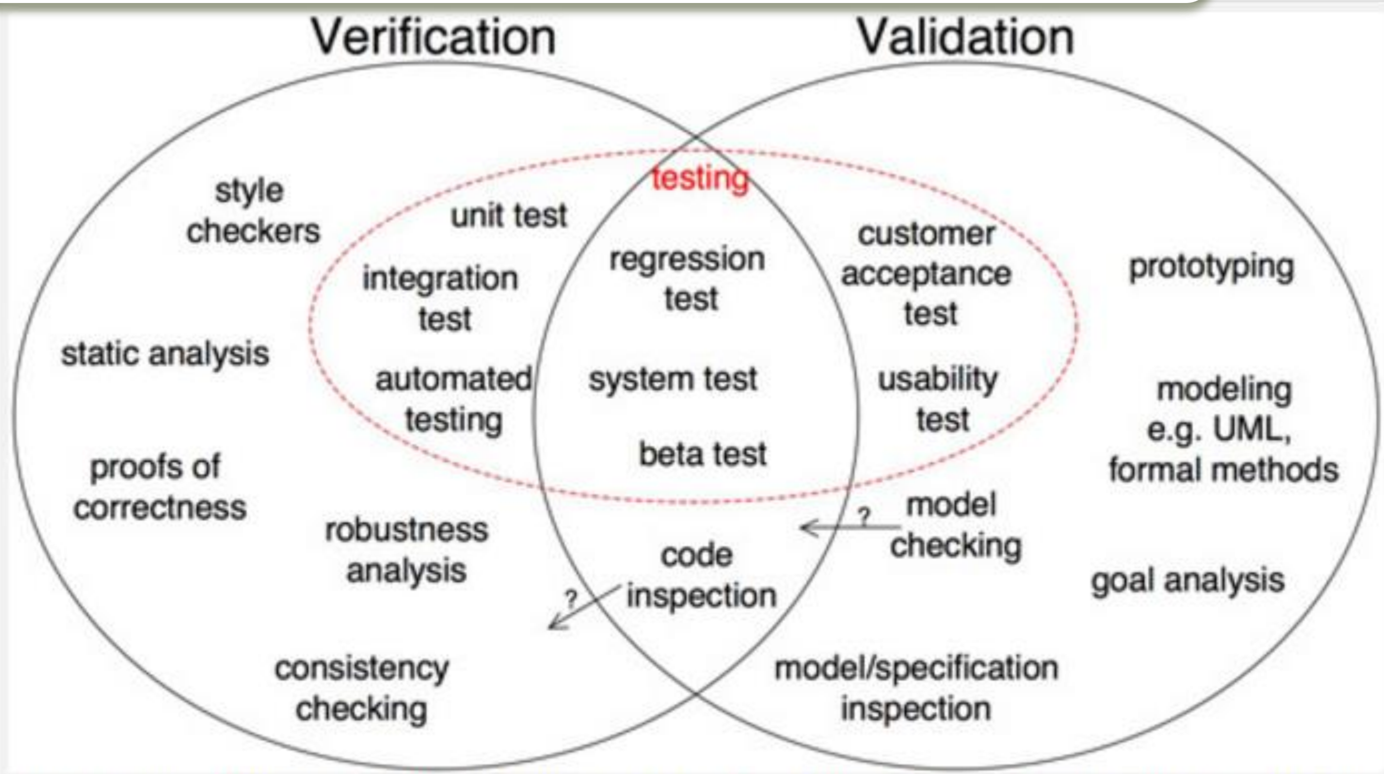

Image Source: http://www.easterbrook.ca/steve/2010/11/the-difference-between-verification-and-validation/

How different between Verification and Validation ? ☺

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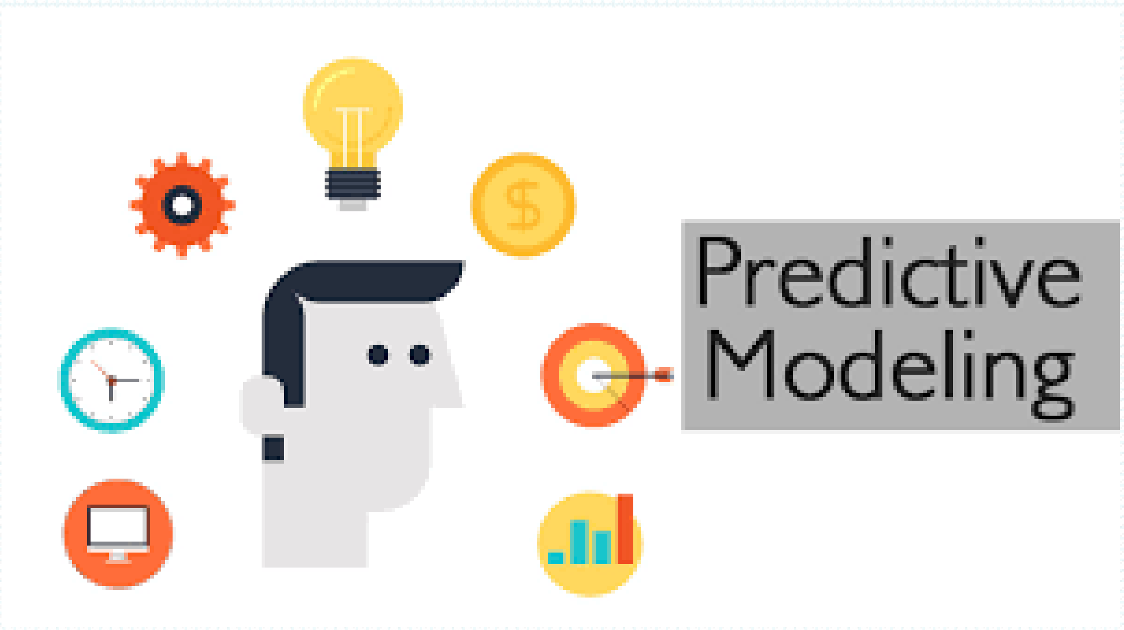
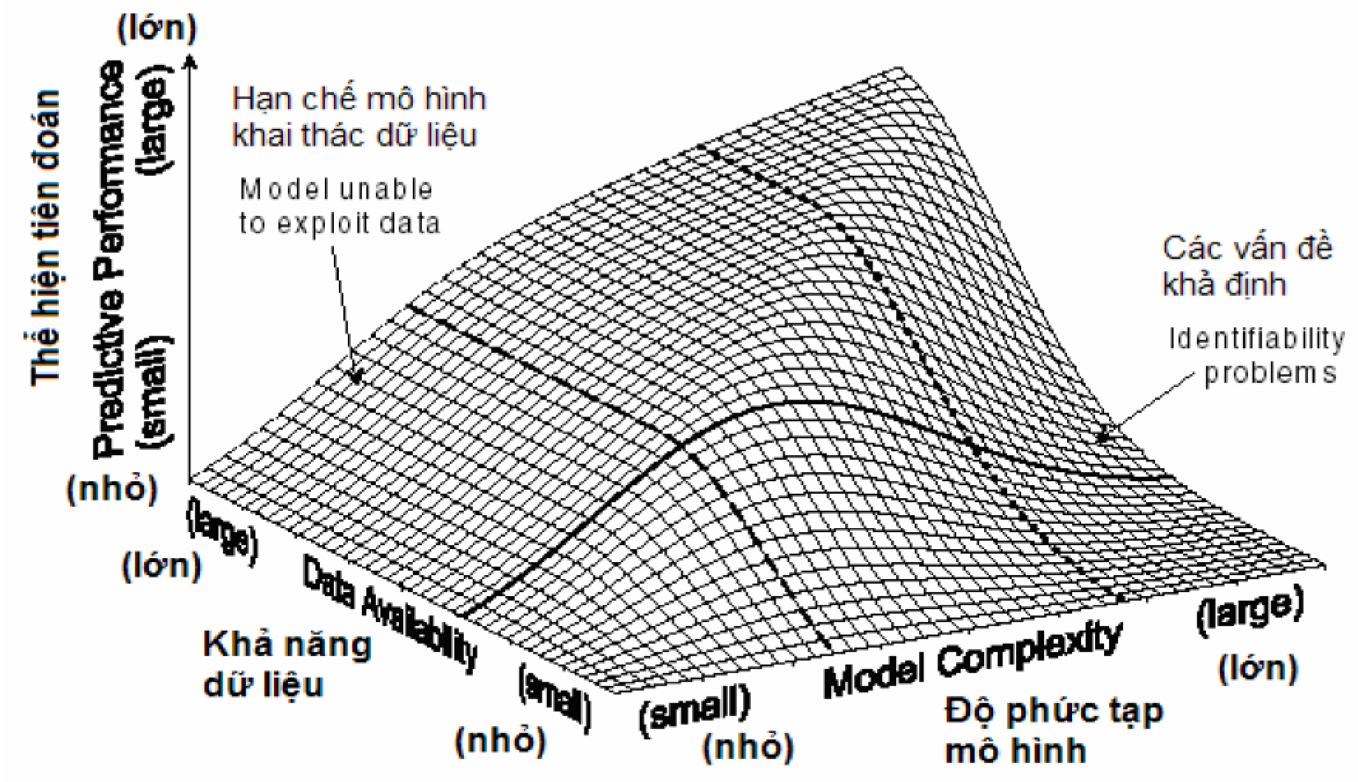
PREDICTION

The model user's goal is to be able to forecast future events by development many scenarios.



CRITERIA FOR MODEL SELECTION



MODEL-PERFORMANCE MEASURES

Part 3

presenting measures are useful for the evaluation of the overall performance of a calibrated, vivificated, predicted model.

The measures may be applied for several purposes, including:

- model evaluation: how good the model is, i.e., how reliable are the model's predictions (how frequent and how large errors may we expect)?;
- model comparison: compare two or more models in order to choose between them;
- out-of-sample and out-of-time comparisons: to check a model's performance when applied to new data to evaluate if performance has not worsened.

Part 3

3.1. Efficiency Index or Coefficient of Efficiency (EI) (Nash-Sutcliffe Efficiency(NSE))

$$EI = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2 - \sum_{i=1}^{n}(X_i - Y_i)^2}{\sum_{i=1}^{n}(X_i - \overline{X})^2}$$

X is observed data,
Y is simulated data

Criteria of Nash-Sutcliffe Efficiency (NSE) Value NSE Value Interpretation

- NSE > 0.75 ➔ Good
- 0.36 < NSE < 0.75 ➔ Qualified
- NSE < 0.36 ➔ Not qualified

Motovilov, Y.G., Gottschalk, L., Engeland, K. dan Rodhe, A. 1999. Validation of a Distributed Hydrological Model Against Spatial Observations. Elsevier Agricultural and Forest Meteorology. 98 : 257-277

MODEL-PERFORMANCE MEASURES

3.2. Correlation Coefficient (R)

$$R = \frac{\operatorname{cov} XY}{s_x s_y}$$

$$\operatorname{cov} XY = \frac{\sum_{i=1}^{n}((X_i - \overline{X})x(Y_i - \overline{Y}))}{(n-1)}$$

- A correlation coefficient of 1 means that for every positive increase in one variable, there is a positive increase of a fixed proportion in the other.
- A correlation coefficient of -1 means that for every positive increase in one variable, there is a negative decrease of a fixed proportion in the other.

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MODEL-PERFORMANCE MEASURES

Part 3

3.3. Standard Deviation(s)

$$S_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$S_y = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

The SD measures the spread of the set of numbers, in effect the average distance of each number from the mean.

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MODEL-PERFORMANCE MEASURES

Part 3

3.4. Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - Y_i)^2}$$

RMSE is NOT scale invariant and hence comparison of models using this measure is affected by the scale of the data → RMSE is commonly used over standardized data.

However, RMSE can be heavily affected by a few predictions which are much worse than the rest

3.5. Mean Absolute Error (MAE)

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|X_i - Y_i\right|$$

MAE and RMSE , Which Metric is Better?

3.6. Root Mean Square Error Mean (RMSEM)

$$RMSEM = \frac{RMSE}{\overline{X}}$$

3.7. Root Mean Square over Standard Deviation (RMSES)

$$RMSES = \frac{RMSE}{s_x}$$

Part 3

3.8. Relative root mean square error (RRMSE)

$$RRMSE = \frac{C_v}{\sqrt{n}}.100\%$$

model accuracy is considered excellent :

RRMSE < 10% ➔ Excellent

10% < RRMSE < 20% ➔ Good,

20% < RRMSE < 30% ➔ Fair,

RRMSE > 30% ➔ Poor

Li MF, Tang XP, Wu W, Liu HB. General models for estimating daily global solar radiation for different solar radiation zones in mainland China. Energy Convers Manag 2013;70:139–48. http://dx.doi.org/10.1016/j.enconman.2013.03.004.

3.9. Mean Percentage Error (MPE) (%)

$$MPE = \frac{1}{n}\sum_{i=1}^{n}(\frac{X_i - Y_i}{X_i})x100$$

3.10. Mean Absolute Percentage Error (MAPE) (%)

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{X_i - Y_i}{X_i}\right|x100 = |MPE|$$

3.11. Percentage of Runoff Volume Error (PVE)

$$PVE(\%) = \frac{Vol^{Y} - Vol^{x}}{Vol^{x}} x100$$

3.12. Time to Peak (TTP)

$$TTP = T_{py} - T_{px}$$

X_i Observed (measured) data at time i

Y_i Computed (predicted) data at time i

n Number of data points

$\overline{X}$ Mean value of observed data $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

$\overline{Y}$ Mean value of computed data $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$

SST Total variation of the value in calibration or verification stage

SSE Sum of the square error

X_{peak} Observed (measured) peak flow data

Y_{peak} Computed (measured) peak flow data

Vol^{X} Observed (measured) runoff volume data

Vol^{y} Computed (predicted) runoff volume data

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Part 4

4.1. Problem 1

The following table shows the measured and computed water level data at station A.

a. Plot (X , Y) in X-axis is Water level (using point and line); number of data points in Y-axis.

b. Compute 9 error parameters/Indices using these measured and computed data.

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Part 4

4.1. Problem 1

Measured Water level (m) - (X_i)	Computed Water Level (m) - (Y_i) (n = 0.33)
2.33	2.23
2.55	2.36
1.75	1.72
1.5	1.45
1.25	1.23
2.35	2.34
2.22	2.11
2.17	2
1.95	1.77
2.15	2

Solution 1

No.	Measured (Xi)	Computed (Yi)	(X_i-Avg X_i)	(X_i-Avg X_i)^2	(Y_i-Avg Y_i)	(Y_i-Avg Y_i)^2	(Xi-Yi)	(Xi-Yi)^2	Abs(X_i-Y_i)	(X_i-Avg X_i)(Yi-Avg Yi)	(Xi-Yi)/Xi
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*	*(12)*
1	2.330	2.230	0.308	0.095	0.309	0.095	0.100	0.010	0.100	0.095	0.043
2	2.550	2.360	0.528	0.279	0.439	0.193	0.190	0.036	0.190	0.232	0.075
3	1.750	1.720	-0.272	0.074	-0.201	0.040	0.030	0.001	0.030	0.055	0.017
4	1.500	1.450	-0.522	0.272	-0.471	0.222	0.050	0.003	0.050	0.246	0.033
5	1.250	1.230	-0.772	0.596	-0.691	0.477	0.020	0.000	0.020	0.533	0.016
6	2.350	2.340	0.328	0.108	0.419	0.176	0.010	0.000	0.010	0.137	0.004
7	2.220	2.110	0.198	0.039	0.189	0.036	0.110	0.012	0.110	0.037	0.050
8	2.170	2.000	0.148	0.022	0.079	0.006	0.170	0.029	0.170	0.012	0.078
9	1.950	1.770	-0.072	0.005	-0.151	0.023	0.180	0.032	0.180	0.011	0.092
10	2.150	2.000	0.128	0.016	0.079	0.006	0.150	0.023	0.150	0.010	0.070
Sum	20.220	19.210		1.506		1.274		0.146	1.010	1.368	0.478
Avg	2.022	1.921									

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Solution 1

No.	Performance Statistics	Values
1	EI	
2	R	
3	Sx	
	Sy	
4	RMSE	
5	MAE	
6	RMSEM	
7	RMSES	
8	RRMSE	
9	MPE	
10	MAPE	
11	PVE	
12	TTP	

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Part 4

4.2. Problem 2

Calibrate C value with measured water level. Try C value between 30 to 60 $m^{1/3}$/s. What is your calibrate C? Compute EI , R and other error parameters which are you choice.

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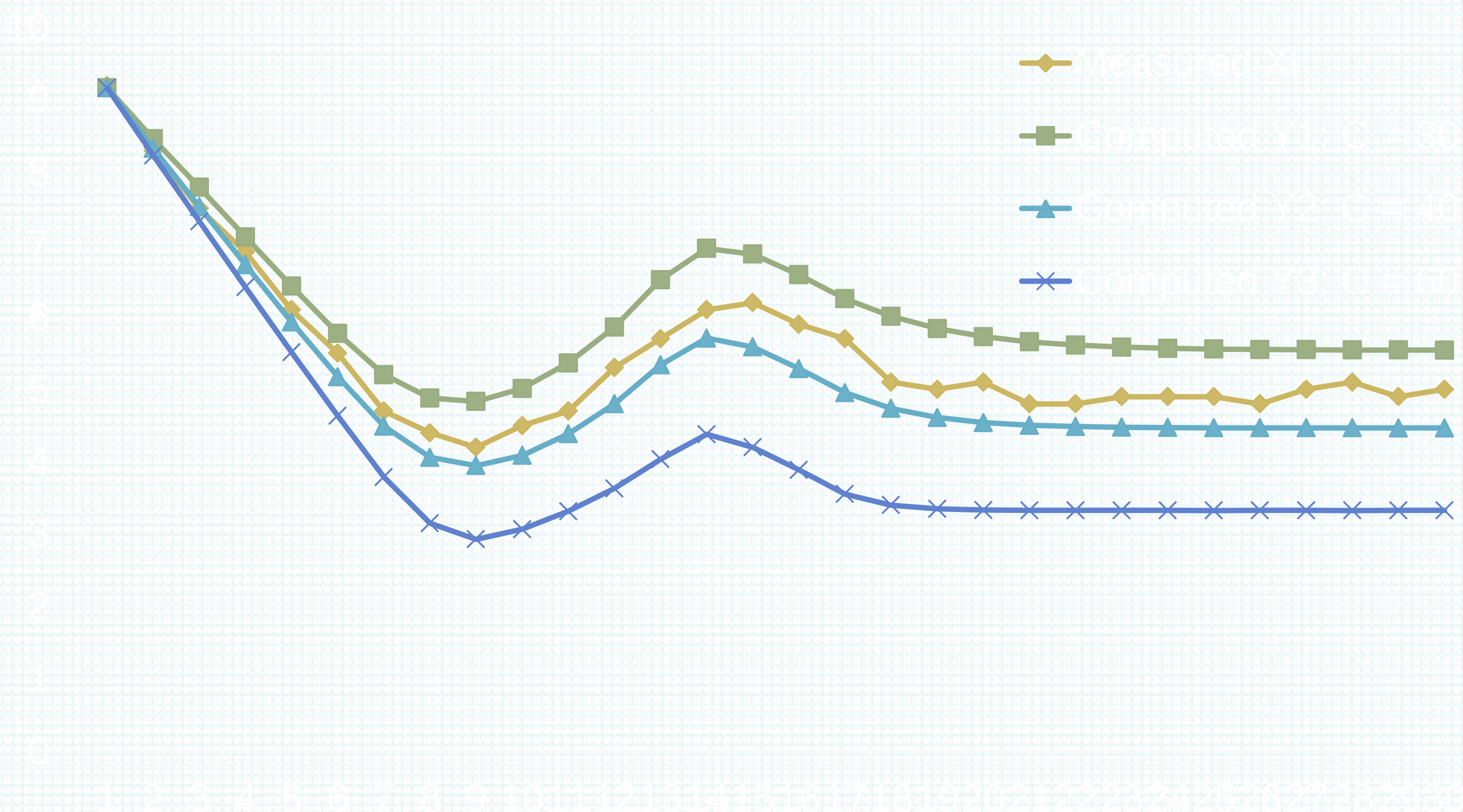
Solution 2

No.	Measured	Computed			No.	Measured	Computed		
	Xi	*Y1: C = 30*	*Y2: C = 40*	*Y3: C = 60*		*Xi*	*Y1: C = 30*	*Y2: C = 40*	*Y3: C = 60*
1	9.2	9.17	9.17	9.17	16	5.9	6.584	5.286	3.886
2	8.3	8.465	8.333	8.235	17	5.7	6.253	4.954	3.553
3	7.5	7.793	7.527	7.323	18	5.1	6.011	4.738	3.4
4	6.9	7.109	6.723	6.413	19	5	5.841	4.611	3.348
5	6.1	6.43	5.933	5.513	20	5.1	5.73	4.54	3.332
6	5.5	5.772	5.174	4.634	21	4.8	5.657	4.504	3.328
7	4.7	5.203	4.492	3.788	22	4.8	5.613	4.486	3.327
8	4.4	4.881	4.061	3.149	23	4.9	5.585	4.476	3.327
9	4.2	4.832	3.946	2.927	24	4.9	5.568	4.472	3.327
10	4.5	5.013	4.088	3.066	25	4.9	5.558	4.469	3.326
11	4.7	5.364	4.386	3.315	26	4.8	5.552	4.468	3.327
12	5.3	5.859	4.799	3.63	27	5	5.549	4.468	3.327
13	5.7	6.516	5.337	4.033	28	5.1	5.546	4.468	3.326
14	6.1	6.95	5.709	4.38	29	4.9	5.545	4.467	3.327
15	6.2	6.871	5.586	4.2	30	5	5.544	4.467	3.327

Solution 2


10
9
8
7
6
5
4
3
2
1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
Measured Xi
Computed Y1: C = 30
Computed Y2: C = 40
Computed Y3: C = 60

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Part 4

4.3. Problem 3

The following table three input number and their old output before modifying and new output after modifying each input by increasing 10% (+10%).

Compute the input sensitivity index for each input number by applying +10%.

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Solution 3

Table 1. Input number 1

Pattern number	Old Input	New Input	Old Output	New Output
1	2	2.2	2.51	4.59
2	5	5.5	9.47	10.7
3	0.8	0.88	1.07	1.2
4	0.5	0.55	0.61	0.45
5	0.5	0.55	0.44	0.35
6	0.5	0.55	0.28	0.3
7	0.5	0.55	0.64	0.65

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Solution 3

Table 2. Input number 2

Pattern number	Old Input	New Input	Old Output	New Output
1	1	1.1	2.51	4.4
2	2	2.2	9.47	10.5
3	2.5	2.75	1.07	1
4	0.2	0.22	0.61	0.5
5	0.2	0.22	0.44	0.4
6	0.2	0.22	0.28	0.25
7	0.2	0.22	0.64	0.7

Solution 3

Table 3. Input number 3

Pattern number	Old Input	New Input	Old Output	New Output
1	1	1.1	2.51	2.0
2	6	6.6	9.47	8
3	3	3.3	1.07	0.2
4	0.9	0.99	0.61	1
5	1	1.1	0.44	0.2
6	0.1	0.01	0.28	0.4
7	0.2	0.02	0.64	0.6

Solution 3

Pattern number	Old Input	New Input	Old Out Put	New Output	(3)-(2)	(5)-(4)	(7)/(6)	(8)^2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	2	2.2	2.51	4.59				
2	5	5.5	9.47	10.7				
3	0.8	0.88	1.07	1.2				
4	0.5	0.55	0.61	0.45				
5	0.5	0.55	0.44	0.35				
6	0.5	0.55	0.28	0.3				
7	0.5	0.55	0.64	0.65				
								Sum=

Similar for Table. Input number 2 and 3

To Select max (col 9) in three tables

Calculate Input Sensitivity Index (IST) = each col 9/Max (col 9)

➔ ISI1, ISI2, ISI3

Plot to graph (ISI in x-axis, input number in y-axis)

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4.4. Problem 4

The table 1 shows three input numbers and their old output before modifying and new output after modifying each input number by increasing 10% (+10%). Table 2 shows the same three input number and their output before modifying and new output after modifying each input number by decreasing 10% (-10%).
Compute the input sensitivity index for each input number by applying +10% and -10%

Solution 4

Table 1

Pattern number	Old Input 1	Old Input 2	Old Input 3	Old Output	New Output 1	New Output 2	New Output 3
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	2	1	1	2.51	4.59	4.4	2
2	5	2	6	9.47	10.7	10.5	8
3	0.8	2.5	3	1.07	1.2	1	0.2
4	0.5	0.2	0.9	0.61	0.45	0.5	1
5	0.5	0.2	1	0.44	0.35	0.4	0.2
6	0.5	0.2	0.1	0.28	0.3	0.25	0.4
7	0.5	0.2	0.2	0.64	0.65	0.7	0.6

Table 2

Pattern number	Old Input 1	Old Input 2	Old Input 3	Old output	New Output 1	New Output 2	New Output 3
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	2	1	1	2.51	3.2	1	2.9
2	5	2	6	9.47	9.5	6	8
3	0.8	2.5	3	1.07	1	1	2
4	0.5	0.2	0.9	0.61	0.7	0.5	0.5
5	0.5	0.2	1	0.44	0.5	0.3	0.1
6	0.5	0.2	0.1	0.28	0.7	0.1	0.4
7	0.5	0.2	0.2	0.64	1.4	0.7	1

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Solution 4

Table 3

Pattern number	Old Input	New Input	Old OutPut	New Output	(3)-(2)	(5)-(4)	(7)/(6)	(8)^2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	2	1.8	2.51	3.2				
2	5	4.5	9.47	9.5				
3	0.8	0.72	1.07	1				
4	0.5	0.45	0.61	0.7				
5	0.5	0.45	0.44	0.5				
6	0.5	0.45	0.28	0.7				
7	0.5	0.45	0.64	1.4				
								Sum=

Similar for Table. Input number 2 and 3

To Select max (col 9) in three tables

Calculate Input Sensitivity Index (IST) = each col 9/Max (col 9) ➔ ISI1, ISI2, ISI3

Plot to graph (ISI in x-axis, Input number in y-axis)

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THE END